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Executive summary

This report analyses the performance and presents potential benefits of the application of Distribution System State Estimation (DSSE). Measurements carried out within Engineering Instrumentation Zones (EIZs) of Low Carbon London (LCL) demonstrate that the developed prototype DSSE, through a limited number of optimally placed sensors could robustly estimate voltage and power flows in High Voltage (HV) distribution networks. This work presents one of the pioneering efforts in examining the role and possible application of DSSE in the present and future distribution networks in the UK.

In contrast to the national transmission system, where measurements are widely deployed to provide visibility and support to the real-time control of the transmission system, available measurement infrastructure in HV distribution networks is not sufficient to facilitate real time control, essential for the evolution to the smart-grid paradigm. Thus additional measurements in distribution networks, of appropriate type and location, will need to be established to support the implementation of innovative real time active distribution network management practices necessary to facilitate cost effective integration of low carbon demand and generation technologies in distribution networks.

As the number of distribution network assets is far greater than the number of assets in the transmission system, it becomes clear that full instrumentation of the entire distribution network may not be justified economically. Hence the challenge is to identify the volume of measurements of appropriate types and locations, to be able to establish the state of the system (“network observability”) across different network operating conditions with adequate accuracy. This work demonstrates that DSSE techniques can minimise investment associated with the deployment of measurement infrastructure whilst maximising the network observability and confidence levels in network voltage and power flow profiles, as well as providing the ability to detect and isolate inaccurate measurements.

The scope of our studies includes the following:

- Development of DSSE model to enhance network observability through estimating network voltage and power flows for a balanced HV distribution network;
- Application of DSSE to determine the optimal number and locations of new sensors;
- Analysis on the application of DSSE to estimate voltage and active and reactive power flows for peak demand condition on the selected Engineering Instrumentation Zones feeders including feeders BRXB-SE3, and BRXB-NE2 from Brixton EIZ, feeder MERT-E2 from Merton EIZ, and feeder AMBL-NW1 from Queen’s Park EIZ;
- Rigorous testing of DSSE model and analysis on the accuracy and robustness of voltage and power flow estimates using half-hourly data across one year;
- Development of an approach to detect bad data and the use of pseudo measurements as alternative for the missing real measurement data; and
- Meter placement studies along the selected EIZ feeders.
Comprehensive sensitivity analyses have also been carried out to assess the impact of uncertainty in the accuracy of the measurements and network parameter data on establishing the state of the system across large number of different network operating conditions.

The key findings from this study can be summarised as follows:

- The DSSE in combination with limited number of sensors and more extensive use of pseudo-measurements\(^1\) is a robust tool for improving the observability of distribution networks, which is increasingly relevant for the present distribution system and critical for the operation of future smart-grid. The DSSE can bring benefits across a number of relevant areas:
  
  - Improve observability in part of the network where the real measurements are not present; there are conditions where new measurements cannot be installed due to space restriction in the substations, especially underground substations;
  - Identify faulty measurements, or temporarily bad data due to recording or communication failures for example, so that the bad quality data can be isolated to improve the overall quality of estimation;
  - Improve the accuracy of the measurements as DSSE takes into account data from all measurements and therefore is able to correct errors from individual measurements;
  - By improving network observability, DSSE could enhance real time distribution network operation activities that may involve network reconfiguration and restoration, support asset management, inform network planning and network pricing;
  - Carry out general assessment of the meter performances, identifying those that should be replaced; and
  - Minimise the cost of monitoring system by reducing the number of sensors needed while being able to estimate voltage and power flow profiles with acceptable accuracy.

- The proposed and developed meter placement methodology in EIZs is robust and its applications have been demonstrated. We have demonstrated in the meter placement studies, that placing 2 or 3 meters have significantly improved the network visibility.

- Our studies suggest that the uncertainty (error margin) of the estimated voltages is relatively small and in most of the cases, the error margin is less than 0.22\% (in comparison to the error margin of individual meters: 0.3\%-0.6\%). We demonstrate that the materiality of the accuracy of power flow estimates will be driven be the capacity/rating of network. The use of DSSE can also enhance network visibility for the lateral sections where the substations may have no measurements installed.

\(^1\) Pseudo measurements are the approximations of the quantity when real measurement data are not available. The pseudo measurement is derived heuristically by using available relevant data, historical data, or any other quantifiable network information associated with the input data in question.
The way in which the available network data is recorded and stored in the Distribution Network Operator’s (DNO’s) database can affect the effectiveness and accuracy of DSSE. For example:

- Some network databases only provide information associated with the first section of the HV feeders, although the rest of the sections could involve cables of different types, having different electrical characteristics. The impedances of the line connecting two nodes are calculated according to the first segment of the feeder which may introduce error in the network parameters and eventually in the DSSE calculations;
- Time synchronisation in taking the measurement samples at different measurement locations is very important for the implementation and application of DSSE. As the system states change dynamically, by using non-synchronous measurement data for the calculation of DSSE increases the error margin of the DSSE output. This is an area that will require further development;
- Bad data and missing data during some time periods can additionally contribute to the inaccuracy of DSSE; and
- Synergies between substation sensors and smart meters could not be used at present, since only a small number of customers at the considered feeders are instrumented.

Unlike in the transmission system where the loading across different phases is balanced, the level of imbalance in distribution network can be large, which would reduce the accuracy of the DSSE which assumes that the system is balanced.

Further work will be required to quantify the economic benefits of the application of DSSE in distribution networks, although it can be preliminary concluded from our studies that the implementation of DSSE will have positive impacts on DNO business activities, e.g. daily operation, planning, asset management, safety and network pricing activities. The benefits of DSSE in reducing the number of measurements required also provide strong business cases for its implementation.

Recommendations

Based on the experience gained in this project, we list a set of recommendations that will assist implementation and application of DSSE in distribution networks. The implementation of DSSE as an integral part of the DMS monitoring system will enhance the capability of the distribution network operator to make informed operation decisions so that the network can be operated securely while making full use available network capacity. To start with, it is recommended that DSSE is employed in stressed parts of distribution networks that are operated close to the operating voltage and/or thermal limits, and then be subject to close monitoring. In order to facilitate the application of DSSE, additional measurements may need to be installed. The approach described in this report can be used to determine the locations and types of measurements needed. Enhancing system state visibility will also benefit network planners, as they can identify the changes in the utilisation patterns and devise optimal strategy for network reinforcement, based on the information gathered from the monitoring system.
It is important to highlight that the main barriers for effective implementation of state estimation lie in the availability and quality of network data. Improvement and standardisation of measurement and recording practice, as well as further enhancement of the DSSE algorithm will contribute to more effective and efficient distribution network system monitoring and control. In this context, our recommendations can be summarised as follows:

- Synchronising readings of all measurement points;
- Improving the availability of key measurements. For example, some of the important measurements are those at the beginning of each feeder; making them available and avoiding the need for pseudo measurements could crucially improve the estimation;
- Checking the accuracy of the recorded network parameters (especially the ones that consist of multiple cables) and the accuracy of recorded transformer winding ratios and the tap-changing positions;

  Validating the measurements accuracy of the sensors in the key feeders where the DSSE is to be applied for operational purposes, as the sensor accuracy is a key factor affecting the outcome of the DSSE. A robust procedure of Remote Terminal Units (RTUs) installation and commissioning needs to be implemented;

- The use of typical load profiles of connected customers in the applications of pseudo-measurements for unmonitored substations is recommended, since it can significantly improve the accuracy of the DSSE model;

- In most cases, placing measurements at feeder supply point at primary substation and towards the end of important feeder branches would enhance the accuracy of voltage estimation and enhance bad data detection.

- As the network imbalance impacts the accuracy of State Estimation (SE), it may be appropriate to consider developing DSSE specifically for unbalanced three phase networks. The application of techniques for improving phase balance, that would reduce network losses and enhance the utilisation of LV and HV networks, would also enhance the accuracy of state estimation;

- A further theoretical development of DSSE, meter placement techniques and algorithms could also be undertaken to develop capabilities addressing network re-configurations on a smartly controlled distribution system environment; and

- Carrying out comprehensive studies analysing and quantifying in more detail the cost and benefits for rolling out the applications of DSSE and to establish standards for its implementation.
## Glossary

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>CT</td>
<td>Current Transformer</td>
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<tr>
<td>DG</td>
<td>Distributed Generation</td>
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<tr>
<td>DER</td>
<td>Distribution Energy Resources</td>
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<tr>
<td>DLR</td>
<td>Dynamic Line rating</td>
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<tr>
<td>DNO</td>
<td>Distribution Network Operator</td>
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<tr>
<td>DS</td>
<td>Distribution System</td>
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<tr>
<td>DSSE</td>
<td>Distribution System State Estimation</td>
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<tr>
<td>EIZ</td>
<td>Engineering Instrumentation Zones</td>
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<tr>
<td>EV</td>
<td>Electrical Vehicles</td>
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<tr>
<td>HP</td>
<td>Heat Pumps</td>
</tr>
<tr>
<td>HV</td>
<td>High Voltage (1 kV ≤ HV ≤ 11 kV)</td>
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<tr>
<td>I&amp;C</td>
<td>Industrial AND Commercial</td>
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<tr>
<td>LCL</td>
<td>Low Carbon London</td>
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<tr>
<td>LCT</td>
<td>Low Carbon Technology</td>
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<tr>
<td>LV</td>
<td>Low Voltage (LV ≤ 1 kV)</td>
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<tr>
<td>NOP</td>
<td>Normally Open Point</td>
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<tr>
<td>ODS</td>
<td>Operational Data Store</td>
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<tr>
<td>OFGEM</td>
<td>Office of Gas and Electricity Markets</td>
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<tr>
<td>RMS</td>
<td>Root Mean Square</td>
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<tr>
<td>RTU</td>
<td>Remote Terminal Units</td>
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<tr>
<td>SE</td>
<td>State Estimation</td>
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<tr>
<td>Tx</td>
<td>Transformer</td>
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<tr>
<td>VT</td>
<td>Voltage Transformer</td>
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<tr>
<td>WLS</td>
<td>Weighted Least Square</td>
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1 Introduction

1.1 Background

Increased penetration of Low Carbon Technologies (LCTs) in the forms of low carbon distributed generation and flexible demand technologies such as electric vehicles, smart appliances, smart homes/buildings and distributed storage, has started to drive the evolution of traditional passive distribution networks into modern smartly controlled distribution networks. In a passive distribution network, the increased penetration of LCTs will eventually trigger inefficient and costly network reinforcement. There is an alternative approach that allows more efficient Distribution Energy Resources (DER) integration to distribution networks, i.e. smart-grid. A smart-grid control approach is effectively changing the conventional operational doctrine from passive to active. Various degrees of integration are possible, ranging from a simple local-based LCT control, to a coordinated control between distribution facilities and LCTs over interconnected distribution circuits. This co-ordinated system-level voltage and flow control could be based on an advanced controller that allows integrated operation to be implemented. An illustrative example is shown in Figure 1.

![Figure 1 An illustrative example of a smartly controlled active distribution network](image)

As illustrated in Figure 1, a Distribution Management System (DMS) with its corresponding telecommunication, monitoring and control functions is critical for smart distribution networks.

Traditionally, the main functions of DMS are to provide network visualisation, and some analytical applications to support network connectivity analysis, load flow analysis, switching schedule and safety management, fault management and system restoration, load balancing and load shedding applications, and distribution load forecasting. Most of the operating decisions are associated with network asset
management (e.g. maintenance) and network re-configuration. While maintaining the traditional functionalities, the future DMS will have much more enhanced functionalities and the ability to optimize the control actions of LCTs and network control devices in a coordinated manner in real time. The future DMS will need to have advanced functionalities for example the abilities to:

- extract the relevant information from a high volume of measurement and control data;
- optimise demand and generation response in order to achieve various network objectives and provide system services;
- predict demand response under various conditions;
- coordinate between distributed / automatic control and centralised control;
- coordinate between preventive and corrective network control;
- control various distributed generation technologies; and
- Manage resources located in distribution network to support transmission system operation (Virtual Power Plant functionality).

In the example above, the DMS controller takes the following information as inputs: (i) measurements of active and reactive network flows and voltages \( (P, Q, V) \), (ii) contract costs for constraining generation on and off, and (iii) network topology, i.e. the states of switches in the network. Its architecture is illustrated in Figure 2. Where there are no local measurements available, the network conditions are assessed using the controller’s DSSE module.

At present, around half of the UK Power Networks (UKPNs) secondary substations in London are equipped with RTUs which record a limited set of determinants [1]. Improved observation of network determinants can be achieved with network monitoring and DSSE techniques. DSSE can also be used to provide pseudo measurements where a network is not instrumented, and detect faulty sensors in more fully instrumented networks.

Based on the information from the DSSE unit, the control scheduling unit optimises the control actions in real time to minimise the operating cost while ensuring that the system is operated securely within the statutory limits. Control instructions issued by the DMS controller include for example: transformer tap positions, LCT schedules \( (P, Q) \), voltage control actions and switching actions.
As the optimality of operating decisions depend on the quality of input information required by DMS control algorithms, it is important to highlight that a successful implementation of DSSE is one of the key components of successful smart grid operation.

In contrast to a transmission system, where measurements have been widely deployed to provide visibility and support to the real-time control of the transmission system, available measurement infrastructure in distribution systems is not sufficient to facilitate evolution to a smart-grid paradigm. For example, at present only feeder-current measurements are typically installed at primary substations and other types of measurements, which are required for SE, such as voltage and power (active and reactive) are not available. Thus, additional measurements in distribution networks, of appropriate type and locations, will need to be determined to support the real time control of the system. In this context, DSSE can be applied to determine the minimum number of measurements needed to establish the state of the system across different network operating conditions with sufficient accuracy and robustness.

Although having a central role in the operation of the transmission system for some decades [2], applications of DSSE in the distribution systems are yet to be established. Thus the performance of the DSSE algorithms in distribution networks that have different and more uncertain network characteristics, e.g. less interconnectivity and to some extent having a higher degree of uncertainty in network parameters and loading, has not yet been fully understood. In this study, carried out for the LCL project\(^2\), the objective is to gain experience on implementing DSSE on real distribution systems and understand its performance, as well as identifying any practical issues and to propose a set of recommendations.

Some trials have been taken place at locations spread all across London, but LCL has also installed additional monitoring equipment on the network within the three Engineering Instrumentation Zones (EIZs) in Queens Park, Brixton, and Merton. Each EIZ comprises a small number of entire HV feeder circuits together with all the secondary substations and Low Voltage (LV) feeders supplied by these circuits. Additional monitoring equipment has been installed at the remote ends of the LV feeders to better understand the performance of the LV distribution network.

1.2 Objective

In this report, the analysis and key results of the performed studies in examining the applications of DSSE in Low Carbon London EIZs are discussed. Based on the work done, this report aims to:

- describe the purpose of traditional distribution network monitoring scheme, the existing state with regards to instrumentation ‘network visibility’ and what this means in terms of understanding network power flows and constraints;
- describe what additional information that future operating paradigms might require, the drivers for it and how this might be delivered;

\(^2\) LCL is an OFGEM Low Carbon Network Fund Tier 2 initiative which is conducting trials and publish research findings across a wide range of topics regarding smart grid implementation and management including the implementation of DSSE. The LCL Programme is a partnership led by UK Power Networks.
• describe the development of DSSE algorithm software using the selected LCL network topology and parameters; and
• Identify to what extent DSSE can reliably substitute network instrumentation.

This study also identifies generic rules that could be used for sensor placement studies and provides a set of recommendations on the number, type and placement location of measurement equipment in order to enable successful implementation of network state estimation.

1.3 Scope

Our analyses cover the results of the applications of the DSSE model developed by Imperial College London on the selected HV feeders from the EIZ including feeder BRXB-SE3, and feeder BRXB-NE2 from the Brixton zone, feeder MERT-E2 from Merton zone, and feeder AMBL-NW1 from the Queen’s Park zone.

The scope of our studies includes the following:

- Development of DSSE model and the optimal meter placement algorithm for a balanced HV distribution network;
- Analysis on the application of DSSE to estimate the voltage and the power flows for the peak demand condition on the selected EIZs feeders including feeders BRXB-SE3, and BRXB-NE2 from Brixton EIZ, feeder MERT-E2 from Merton EIZ, and feeder AMBL-NW1 from Queen’s Park EIZ;
- Rigorous testing of DSSE model and analysis on the accuracy and robustness of voltage and power flow estimations using one year half-hourly data;
- Development of an approach to detect bad data and the use of pseudo measurements as alternative for the missing real measurement data; and
- Meter placement studies along the selected EIZ feeders.

Furthermore, we have carried out a set of sensitivity studies to analyse the impact of uncertainty in the accuracy of the measurements and network parameter data.

1.4 Structure of the report

The report is structured as follows:
- Chapter 2 discusses the DSSE methodology used in this study, the input data requirements and assumptions to fill the gap between the available data and the input data required by the DSSE model. Furthermore, it describes some practical issues found while carrying out the studies;
- Chapter 3 and 4 describe the key results and our analysis on the applications of the DSSE model on the selected EIZ feeders;
- Chapter 5 discusses the approach for optimal meter placement and the application of the developed approach on the selected EIZ feeders; and
- Chapter 6 contains the conclusion and some recommendations based on the results of those studies.
2 Methodology and the Application of DSSE Model to the Engineering Instrumentation Zones

2.1 Distribution System State Estimator

The basic task of a distribution system state estimator is to estimate as accurately as possible the true state of a system (i.e. voltage and power flow profiles), using any relevant available information. In a typical state estimator, a state of the system is represented by a set of voltage magnitudes and voltage angles. This allows power flows and current along each feeder to be derived. The DSSE model developed for this purpose is based on the maximum-likelihood estimation of state variables which employs Weighted Least Square (WLS) formulation, which is recognised as one of the most suitable techniques for state estimation in distribution systems [3].

2.1.1 Basic formulation

The general SE problem is to identify the true system state from available information of measurements which generally have certain inaccuracy and can be formulated as follows:

\[ z_{\text{measured}} = z_{\text{true}} + \text{error} \]  

Where \( z_{\text{measured}} = [z_1, z_2 \ldots z_m]^T \) is a vector containing the values of all \( m \) measurements in the considered system (such as voltages and power flows on different measurement points);
\( z_{\text{true}} \) is a vector of the true values of system state, as they would be if there were no errors in measurements and communication links, and
\( \text{error} \) is a vector representing the differences between the true and measured values, influenced by the uncertainty in measurement devices and errors in the communication system.

Each value of the vector \( z_{\text{true}} \) can be formulated as a function of state variables. The model contains a set of nonlinear functions governing the relations between the measurements and state variables (voltages and angles) at all nodes, that can be formulated as:

\[ z_{\text{measured}} = h(x) + e \]  

Where \( x = [\theta_1, \theta_2 \ldots \theta_n, V_1, V_2 \ldots V_n]^T \) is the state variables vector, comprising voltage magnitudes and angles (\( n \) - number of busses);
\( h(x) \) is a vector of known functions relating state variables and values whose quantities are measured (such as voltage, power injections and power flows), and
\( e = [e_1, e_2 \ldots e_m]^T \) is a vector containing the errors\(^3\) attached to each measurement. The measurement errors are considered to have zero mean Gaussian noise, with measurement error covariance matrix

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\(^3\) Error in measurement may be represented by a tolerance interval (margin of error). Machines used in manufacturing often set tolerance intervals, or ranges in which product measurements will be tolerated or accepted before they are considered flawed.
\( R = \text{diagonal} \{ \sigma_1^2 \sigma_2^2 \ldots \sigma_m^2 \} \). It consists of variances of measurements equal to \( \sigma_i^2 \), each reflecting the expected accuracy of the metering device.

The problem is to find the state vector \( \mathbf{x} \) which best fits equation (2-2). The method used to achieve this is the WLS formulation, where the task is to minimise difference between measured and true values taking in consideration weight of each measurement represented by its variance. The DSSE model employs a Newton iterative technique to solve SE’s non-linear equations [2].

### 2.1.2 Input data for the DSSE model

The input data required by the DSSE model are listed as follows:

- **Network topology** - the single phase positive sequence equivalent circuit is used to model the network under study assuming steady state balanced conditions;
- **Network parameters** – such as impedance, susceptance and tap changer positions of transformers;
- **Measurements:**
  - Real measurements – measurements of voltage magnitudes, active and reactive power injections and active and reactive power flows. Data accuracy depends on the accuracy of the measuring equipment and communication system.
  - Pseudo measurements - the approximations of the quantity when real measurement data are not available. The pseudo measurement is derived heuristically by using available relevant data, historical data, or any other quantifiable network information associated with the input data in question. Consequently, the accuracy of pseudo measurement is relatively low. However, this approach is necessary to compensate the lack of real network measurements and to ensure the solvability of the DSSE mathematical model. The way the pseudo measurements are modelled has an important impact on estimation quality.
  - Virtual measurements – for example it is assumed no power injections at the joint points in the network.
- **Weighting factor of the measurements** - The higher the meter’s accuracy, the larger the weight. This implies that the algorithm gives more importance to the accurate measurements. The probability distribution of the measurement errors is assumed to follow normal or Gaussian distribution. We assume that the maximum error tolerance specified for each sensor/meter covers 99.7% of the area of the Gaussian curve. Thus, the standard deviation (\( \sigma \)) of the measurement error can be formulated as follows:

\[
\sigma = \frac{\text{Mean(Measurement)} \times \text{Error(Measurement)}}{3 \times 100}
\]

Information about meter’s accuracy is very important since it defines the weighting factor of each measurement. Weighting is inversely proportional to the variance of the measurement (\( \sigma^2 \)).
For the purpose of this work the following values of measurement errors are assumed:
- Error of the virtual measurements, at joint nodes, is assumed to be 0.001%;
- Error of the Pseudo measurements, P and Q at substations with no measurement devices, is assumed to be 50%;

From the meters installed in primary and secondary substations:
- Primary Substation:
  - Error in voltage measurement devices: 1%
  - Error in active and reactive power measurements: 2%
- Secondary Substation
  - Error in voltage measurement devices: 0.3%
  - Error in active and reactive power measurements: depends on the current through CT and goes up to 9.6%. From the test results for the clips on CTs - the mean error dependency on current through CT is found, as illustrated in Figure 3. The CTs are rated for 1000A but the measurement current is rarely above 500A (please note the current is measured on the LV side of the transformer). This is applied for calculating the measurement error of active and reactive power.

\[
\gamma = 0.0000151850x^2 - 0.0248535072x + 9.6338510237
\]

Figure 3 Measurement error dependencies on current through CT

### 2.2 Input data assumptions

There is a set of assumptions used in the DSSE model. These include:
- Topology data are known and accurate;
- Parameters data, such as resistance and reactance of all lines, resistance, reactance and tap changer position of all transformers are known and accurate;
- Measurements data and accuracy of the measured unit are available:
  - Voltage magnitude at each node of the feeders,

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\( ^4 \) Information from UK Power Networks, supported by the RTU manual
o Total reactive and active power at the supply point of the feeder,
o Reactive and active power at each secondary substation;
- Measurements recorded in a consistent manner;
- Full sensors coverage of the network; and
- The system is a balanced three-phase system.

2.2.1 Output data of the DSSE model

The output data of DSSE model are as follows:

- primary variables - representing estimates of state variables which comprise voltages and voltage angles, and state error covariance matrix $P_x$, which indicates the accuracy of the estimates;
- The secondary variables and their covariance matrix - which are the quantities (e.g. active and reactive power injections and power flows) derived from the estimated state variables and state error covariance matrix.

2.3 Application of state estimation to the Engineering Instrumentation Zones

One of LCL trial objectives was to fully instrument the selected four 11kV feeders from three EiZs, in order to have solid test platforms for this work. These four feeders are: BRXB-SE3 and BRXB-NE2 from the Brixton zone, MERT-E2 from the Merton zone and AMBL-NW1 from the Queen’s Park (alias Amberley) zone.

For the purpose of the studies it was envisaged to have various input data including network topology, network parameters of each network element and measurements in the primaries of considered feeders and each secondary substation. This data have been stored in Operational Data Store (ODS) of the UK Power Networks. Due to the necessity of compression, the values of the measurements at the primary substations were recorded once when the time threshold or the change-in-value threshold had been reached. Consequently, the time difference between 2 recordings varies ranging from seconds to several hours. This issue is discussed further in the next section.

The secondary (11/0.4 kV) substations are mainly instrumented with the RTUs. These measurements utilise current transformers (CT) at the LV side of the 11kV/0.4kV transformers. The RTUs record the measurements and derive the average rms values on a half-hourly basis. For the purpose of this project, new RTUs which have higher data resolution and record the average rms measurements every 10 minutes have been placed at accessible substations. Due to practical issues some substations could not be fully instrumented. For example, the underground substations with limited space or there was inadequate GPRS coverage to allow data transfer.

A summary of the available and the required measurement data from primary and secondary substations is given in Table 1. The detailed analysis on accessibility and validity of substation sensor data is given in [4].
Table 1 Summary of the available and the required measurement data from primary and secondary substations

<table>
<thead>
<tr>
<th>Zones</th>
<th>Queens park</th>
<th>Merton</th>
<th>Brixton</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary Substation Transformers Measurements</strong></td>
<td>~</td>
<td>A,kV,MW, MVAr</td>
<td>A,kV,MW, MVAr</td>
</tr>
<tr>
<td><strong>Feeder supply point measurements</strong></td>
<td>AMBL-NW1</td>
<td>MERT-E2</td>
<td>BRIXB-NE2</td>
</tr>
<tr>
<td>Required V, P, Q</td>
<td>~</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Existing V, P, Q</td>
<td>~</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td><strong>Secondary substations measurements</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Required V, P, Q</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Existing V, P, Q</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Secondary substation HV measurements</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Secondary Subst.</td>
<td>4</td>
<td>13</td>
<td>7</td>
</tr>
<tr>
<td>No. Of Sub. with measurements</td>
<td>3</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>No. of Sub. with valid measurement data</td>
<td>3</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

2.4 Practical issues

The existing distribution network monitoring system has been designed with the objective to provide network visibility which traditionally is used only for asset management and meeting safety regulation with minimum measurement infrastructure required. While this is not an issue in passive distribution networks that mostly have sufficient capacity margin to deal with changes in load, in an actively managed network, the inadequacy of the existing scheme becomes more apparent. As voltages and power flows across the network need to be monitored more closely in the actively controlled distribution network environment, the insufficient network coverage, bad quality of measurements data, uncertainty in network parameters and imbalanced load, can cause the DSSE model fail to provide useful results. These issues are elaborated in more detail as follows:

- **Parameters accuracy** – The cables connecting two nodes of the network usually consist of multiple segments. Very often these segments are made of different types of cable, having different electrical characteristics. The impedences of the line connecting two nodes are often calculated only based on the impedances of the first segment of the branch. This increases the level of inaccuracy in recording the real network parameters. While it is difficult to determine precisely by how much this inaccuracy will affect the results of DSSE model since it depends on the level of differences between the cable parameters in question, the use of inaccurate network impedance and susceptance will certainly reduce the accuracy of the DSSE model.

- **Measurements recorded quantity and quality** – Currently there is only partial network coverage, as detailed in Table 1.

  o **Unavailable V, P and Q measurements at the supply points.** There are only records of current measurements at the supply points of 3 feeders. Moreover, Queen’s Park EIZ does
not have any recording on the primary substation. V, P and Q are measured in primary transformer circuits.

- The measurements on secondary substations are available only on 24 locations out of 34. However, only 21 are deemed to be valid as will be detailed in the following chapters.

**Measurements recording sample resolution.** For the purpose of SE, the recording of various measurements should be synchronised, which was not always achieved. We have also observed the differences in the recording practices: at a primary substation, instantaneous values of measurements are recorded while at secondary substation, average values are stored. For this purpose, we apply data interpolation where required (further discussion in section 2.4.1).

- **Bad data and missing data** that may reduce the accuracy of SE.

- **Inadequate number of installed smart meters to enable synergy between substation sensors and smart meters.** Once, smart meters are widely rolled out, this opens opportunity to integrate smart meters as components of a distribution network monitoring system.

- **Imbalance affecting results** – the single phase positive sequence equivalent circuit is used to model the networks assuming steady state balanced conditions. In reality the system is not always balanced and this affects the accuracy of SE. This issue will be discussed in more detail in Section 2.4.3. In order to implement DSSE on an unbalanced three-phase system the DSSE model has to be further developed and measurements must be done in all phases.

### 2.4.1 The unavailability of V, P and Q measurement data at feeder supply point

The DSSE model requires measurement for V, P and Q, but they are not provided at the HV feeder supply point, with the feeder current only provided. The current is measured in Amperes on a very frequent basis, but recording depends on the time and value thresholds, which are not clearly defined. In this section, we describe the approach used to fill the gap illustrated for a feeder at Brixton primary substation.

The Brixton primary substation is supplied by four 33 kV/11 kV transformers and their A, kV, MW, MVAr measurements are recorded. These 4 transformers supply 16 HV feeders, and can be interconnected by 4 bus couplers. According to the topology data, the bus couplers are closed during the considered year, giving an impression that all 4 transformers are working in parallel. However, the current measurement through the bus couplers during, for example January, is always equal to 0 for bus coupler BS1-2 and BS3-4, indicating that the pairs of transformers 1 and 4, and 2 and 3 are connected on the HV bus-bars side.

To reconstruct the missing data, the measurement data of the closest transformer (if operating) to the feeder, are used under the following assumptions:

- voltage at the supply point of the BRXB-SE3 feeder is equal to the voltage at the closest 33/11 kV transformer which is transformer T4;
feeder active and reactive power are derived using the following information: current measurement data and T4 voltage to establish the apparent power, and in order to calculate active and reactive power we assume that the power factor of load at the feeder is the same as the power factor of load at transformer T4.

If the closest transformer is not in operating state, e.g. if the voltage is equal to 0 for a certain period, then the next closest transformer is considered as a supplying transformer.

As Figure 4 shows during 31 January 2014, we observe data inconsistency in the values of recorded voltage measurements. Based on the network data we have, we assume that the voltages at the HV side of the transformers in this primary substation must be practically the same. However, based on the recorded values in a number of times, the difference between the voltages of the two connected transformers is more than 3%. This implies that the transformers are not always fed from the same HV node. This discrepancy could increase error in SE.

Since all electrical quantities measured at the primary substation are recorded with varying sampling rates and the secondary substation measurements recorded as half-hourly average values, it was necessary to convert the measurement data from the primary substations to half-hourly average values. Figure 5 shows a sample of measured BRXB-SE3 feeder current and the average value of each 30-min period. Considering that the recording time interval varies significantly, i.e. seconds to hours, such averaging process increases the level of error in our approximations. It is difficult to determine the significance of the error without having the results from the proper (synchronised) measurement system, but it is likely that the extent of the error is capped to the minimum step-change settings used in the measurements.
It is inevitable that the values obtained using the above process will have higher uncertainty compared if those values are directly measured. The installation of new type RTUs in primary substations which can measure more parameters per feeder may provide more accurate values.

### 2.4.2 Impact of an unbalanced three-phase system on State Estimation

The following example illustrates the extent of voltage unbalance at secondary substation 90069, see Figure 6. During 12 January 2014, the highest voltage unbalance was recorded at 18:00, when the difference between the voltages at L2 (yellow) and L3 (blue) phase was 2.38%.
Figure 6 Example of voltage unbalance at substation 90069

Figure 7 illustrates the apparent power at the substation 90044 for a 24 hour period recorded on the 1 October 2012. The black line represents the phase average apparent power, whilst the red, yellow and blue lines show the calculated apparent power of the L1 (red), L2 (blue) and L3 (yellow) phases, using the respective phase voltages and currents. The total value of the calculated apparent power indeed equals the one recorded in the ODS. However, by not recording power per phase, one may jump to the wrong conclusion that the system is three-phase balanced. Since the single phase positive sequence equivalent circuit is used to model the networks under study assuming steady state balanced conditions, the fact that the system is not balanced reduces the accuracy of the DSSE model. While this is a limitation of the current model, applying DSSE to an unbalanced system will also not be possible in this project, taking into account that only phase values of currents and voltages are recorded from the old RTU substations, but not power factor or active and reactive power per phase, and there are no such records from the primary substations.

Figure 7 Example of power imbalance

Figure 8 shows a voltage profile, where measured, at secondary substations of BRXB-SE3 feeder recorded on 31 January 2014.

Figure 8 Example of quality of the substation BRXB-SE3 voltages
The voltage profiles are sorted from the one closest to the supply point - substation 90069 to the farthest 91045. It is expected that voltage drops along the feeder driven by loads as no generation is recorded along this feeder. However we observed data inconsistency. For example, voltage at substation 90043 can be higher than the voltage at substations 90044 and 90069, which are located closer to the primary substation; the voltage at substation 94192 is lower than voltage at substation 91045; substation 90862 has lower voltage than the three farther substations during high loading hours. Since these voltages represent the average value of three phases’ voltages recalculated on the HV side of the transformers, these inconsistencies could indicate that the system is actually unbalanced, i.e. opposite to the assumption taken by the DSSE model. The additional factor which could contribute to these differences could be the recording error of transformers impedances, or position of tap changers (i.e. transformer ratio).

As the network imbalance impacts the accuracy of SE, it may be appropriate to consider developing DSSE specifically for unbalanced three phase networks. Application of techniques for improving phase balance that would reduce network losses and enhance the utilisation of LV and HV networks would also enhance the accuracy of state estimation.

2.4.3 The use of pseudo measurements to substitute missing or invalid measurement data

In the case of feeder BRXB-SE3, RTUs are not available for substations 90625 and 94356, and data from substation 91143 is erroneous. Therefore we used pseudo measurements to substitute the missing or invalid data. The difference between the feeder loading and the known substations loadings are distributed to these 3 substations proportionate to their ratings. Unfortunately, we cannot derive the pseudo measurement data based on the typical load profile of the substations, since the consumption data is incomplete (data for substation 94356 is not available). Moreover, the consumption of the substations was difficult to be derived due to the insufficient amount of LV measurement data. For example, there are only kWh measurements for three industrial and commercial customers out of 143 customers connected to substation 90625. Furthermore, smart meters data cannot be utilised since they are installed only at a small number of LV customers. Once smart meters are widely rolled out there is the opportunity to integrate smart meters as components of a distribution network monitoring system.
3 Application of DSSE for Feeder BRXB-SE3 in Brixton EIZ

3.1 Overview

Feeder BRXB-SE3 is one of two fully instrumented HV feeders in the EIZ supplied from the Brixton primary substation. Its total length is 2.575 km and it supplies around 2000 domestic, industrial and commercial customers via ten 11 kV/0.4 kV transformers at secondary substations. There are two 1 MVA and eight 0.5 MVA rated transformers. Figure 9 shows the diagram of the feeder. Please note that the distances between the secondary substations in the diagram are not proportional to the actual circuit length.

The majority of the distribution transformers are instrumented with RTUs. Solid and dotted red ellipses mark the location of substations where some or all measurement data are not available or not of good quality. For example, substation 90625 is an underground substation and, therefore, unfit for new
sensor installation. The ODS database also does not have measurement records for substation 94356 nor the number of customers it supplies. The dotted red ellipse marks substation 91143, which is equipped with the new type of RTU. However, data measurement from the RTU are insufficient (it is likely due to data recording issues) and erroneous. The blue dotted ellipse marks the primary substation node of feeder BRXB-SE3, where only the currents are recorded.

The list of the nodes with substations’ ratings and number of different classes of customers is presented in Table 2. The explanation of customer class profiles is given in Table A. 1 in the Appendix. The substations with missing measurement data are marked red. The cable ratings under normal conditions are 280 A or 285 A and the maximum recorded current at the supply point of the feeder is 131 A.

Table 2 BRXB-SE3 Substations’ data. Missing measurement data are marked red, and derived measurement data - marked blue

| Index | Node Name | Rating | Total Cust. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------|-----------|--------|-------------|---|---|---|---|---|---|---|---|---|---|
| 1     | Prim.Sub. | 1829   | 4           | 1535 | 44 | 233 | 12 | 0 | 0 | 0 | 1 | 0 |
| 2     | 90069     | 500 kVA| 193         | 1   | 171 | 10 | 10 | 1  | 0 | 0 | 0 | 0 |
| 3     | 90044     | 500 kVA| 66          | 1   | 50  | 1  | 14 | 0  | 0 | 0 | 0 | 0 |
| 4     | 90043     | 500 kVA| 82          | 0   | 53  | 2  | 25 | 2  | 0 | 0 | 0 | 0 |
| 5     | Joint     | 0      | 0           | 0   | 0   | 0  | 0 | 0  | 0 | 0 | 0 | 0 |
| 6     | 90625     | 500 kVA| 143         | 0   | 124 | 3  | 15 | 1  | 0 | 0 | 0 | 0 |
| 7     | 90862     | 500 kVA| 100         | 0   | 16  | 0  | 84 | 0  | 0 | 0 | 0 | 0 |
| 8     | 94356     | 1.0 MVA| 0           | 0   | 0   | 0  | 0 | 0  | 0 | 0 | 0 | 0 |
| 9     | 90638     | 500 kVA| 130         | 0   | 119 | 0  | 10 | 1  | 0 | 0 | 0 | 0 |
| 10    | 91143     | 500 kVA| 191         | 0   | 184 | 0  | 7  | 0  | 0 | 0 | 0 | 0 |
| 11    | 94192     | 1.0 MVA| 585         | 1   | 498 | 25 | 55 | 6  | 0 | 0 | 0 | 0 |
| 12    | 91045     | 500 kVA| 339         | 1   | 320 | 3  | 13 | 1  | 0 | 0 | 1 | 0 |

* there is no information on number of customers supplied from substation 94356, therefore, the specified total number of customers supplied by the feeder does not include those customers.

Network data is presented in Table 3. The cable ratings under normal operating conditions are 280 A or 285 A and the maximum recorded current at the supply point of the feeder is 131 A.

Table 3 BRXB-SE3 Feeder network data

<table>
<thead>
<tr>
<th>ID</th>
<th>from node Index</th>
<th>to node Index</th>
<th>from node</th>
<th>to node</th>
<th>Length (Via OS) (m)</th>
<th>Resistance (R) (% at 100 MVA base)</th>
<th>Reactance (X) (% at 100MVA base)</th>
<th>Line Rating - min(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>90069</td>
<td>Prim.Node.</td>
<td>192.7</td>
<td>2.756</td>
<td>1.301</td>
<td>285</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>90044</td>
<td>90069</td>
<td>229.0</td>
<td>3.320</td>
<td>1.551</td>
<td>280</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4</td>
<td>90044</td>
<td>90043</td>
<td>162.4</td>
<td>2.359</td>
<td>1.099</td>
<td>280</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td>Joint</td>
<td>90043</td>
<td>13.9</td>
<td>0.202</td>
<td>0.095</td>
<td>285</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>6</td>
<td>Joint</td>
<td>90625</td>
<td>131.6</td>
<td>2.192</td>
<td>0.899</td>
<td>280</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>5</td>
<td>90862</td>
<td>Joint</td>
<td>154.4</td>
<td>2.343</td>
<td>1.050</td>
<td>285</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>7</td>
<td>94356</td>
<td>90625</td>
<td>386.5</td>
<td>5.257</td>
<td>2.582</td>
<td>285</td>
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<tr>
<td>8</td>
<td>8</td>
<td>9</td>
<td>94356</td>
<td>90638</td>
<td>288.0</td>
<td>4.027</td>
<td>1.815</td>
<td>280</td>
</tr>
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<td>9</td>
<td>10</td>
<td>9</td>
<td>91143</td>
<td>90638</td>
<td>403.2</td>
<td>6.043</td>
<td>2.592</td>
<td>280</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>10</td>
<td>94192</td>
<td>91143</td>
<td>498.3</td>
<td>7.941</td>
<td>3.399</td>
<td>280</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>11</td>
<td>91045</td>
<td>94192</td>
<td>115.2</td>
<td>1.483</td>
<td>0.751</td>
<td>285</td>
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</tbody>
</table>

Figure 10 shows one-day half-hourly average load profiles of each monitored substations which can be used to identify the type of the customers supplied in this feeder. Residential customers typically have
evening peak load, while commercial and industrial customers’ peak load typically occurs during midday. This suggests that feeder BRXB-SE3 supplies different types of customers.

![Figure 10 Feeder BRXB-SE3 - Averaged yearly daily profile of equipped substations](image)

### 3.2 Results of state estimation application to BRXB-SE3 feeder

In this study, the system state of the BRXB-SE3 feeder is estimated for each half-hourly period from 01 March 2013 until 01 March 2014. Data associated with the accuracy of the measurements used in this study can be found in Section 2.1.2. The key outputs of the DSSE model are the expected values and confidence intervals of voltages, angles, active and reactive demand and active and reactive power flows for each period. These values are used to estimate the measurement errors and for bad data detection and in the optimal meter placement algorithm.

#### 3.2.1 Studies using the peak demand condition

In this example the DSSE model is used to estimate the peak demand condition operating regime of BRXB-SE3. The half-hourly peak demand occurred on the 31 January 2014 at 19:00. Since there were no measurements at nodes 6, 8 and 10, active and reactive power pseudo measurements with low accuracy have been used. The difference between the power, for both active and reactive power, measured at primary substation and the total power at known substations, is distributed proportionally according to their rating to the secondary substations which do not have measurements. The measured data and their errors are presented in Table 4. Pseudo measurement data are highlighted in red.

The DSSE results for voltages are shown in Figure 11. Measured values are denoted with blue squares. The expected values of voltages are represented by red circuits on the thick red line. The red dotted and dashed lines represent the boundaries within which the true value voltage is expected i.e. within ±3 standard deviations of the average estimates. The black dotted and dashed lines represent the measuring equipment error margin of ±0.6% at secondary substations and ±1% at primary substation. It is demonstrated that all measurements are within the equipment accuracy. We observe that the
measured voltages do not follow the expected pattern of a constant drop down the feeder; however, the estimated voltages do follow the expected pattern.

Table 4 BRXB-SE3 feeder’s measurement data - peak demand condition on 31/01/2014 at 19:00:00

<table>
<thead>
<tr>
<th>Node Index</th>
<th>Node Name</th>
<th>V measured</th>
<th>V error %</th>
<th>P measured/pseudo [kW]</th>
<th>P error %</th>
<th>Q measured/pseudo [kW]</th>
<th>Q error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Prim.Sub</td>
<td>0.997</td>
<td>1</td>
<td>2340.67</td>
<td>2</td>
<td>498.11</td>
<td>2</td>
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<td>2</td>
<td>90069</td>
<td>0.989</td>
<td>0.6</td>
<td>142.66</td>
<td>5.46</td>
<td>20.74</td>
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<td>90044</td>
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<td>65.67</td>
<td>7.57</td>
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<td>5.53</td>
<td>14.36</td>
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</tr>
<tr>
<td>5</td>
<td>Joint</td>
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<td>-</td>
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<td>0.001</td>
<td>0</td>
<td>0.001</td>
</tr>
<tr>
<td>6</td>
<td>90625</td>
<td>-</td>
<td>-</td>
<td>177.94</td>
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<td>50</td>
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<td>7</td>
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<td>94356</td>
<td>-</td>
<td>-</td>
<td>355.89</td>
<td>50</td>
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<td>10</td>
<td>91143</td>
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<td>-</td>
<td>177.94</td>
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<td>82.61</td>
<td>50</td>
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<td>367.08</td>
<td>1.11</td>
<td>24.77</td>
<td>1.11</td>
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</table>

Figure 11 Estimated voltages and voltage-measurement data in the studies using peak demand condition
The level of error in estimating voltages is presented in Figure 12. The level of error is relatively small and does not exceed 0.22% across all nodes. Even though the margin of error is similar, the farther the network node is from the primary substation the level of uncertainty increases.

![Figure 12](image)

**Figure 12** Accuracy of the estimated voltages in the studies using peak demand condition

Estimates of active and reactive power demand and the range of estimation error for each node are presented in Figure 13 and Figure 14, respectively. The estimates, red coloured rhombus, are mainly overlapping with the measured values (blue squares) or pseudo values for missing measurements (green rhombus). However the estimates could be in the range between the boundaries of ±3 standard deviations of estimates (red star and x). These ranges are relatively high for unknown measurements.

![Figure 13](image)

**Figure 13** Estimated and measured active power in the studies using peak demand condition
The level of error in estimating active power injection is illustrated in Figure 15. Although the initial error designated for active power of unmonitored substations 90625, 94356 and 91143 was 50%, as listed in the Table 4, the level of error for nodes 90625 and 91143 is slightly smaller. The level of error is equal to approximately 46% for two of substations and around 30% for the substation 94356. More of known measurements with high accuracy can contribute to better estimates of the non-measured loads. This demonstrates the advantage of state estimation, as the approach can improve the accuracy of individual measurements, especially ones with high level of uncertainty, by aligning them with the results of DSSE model that have taken into account information from other measurement data. The similar pattern follows the uncertainty of reactive power.
It is important to note that even the error margin for un-monitored substations (90625, 94356, and 91143) is relatively high, the application of DSSE actually improves the visibility for these substations, which were not previously visible. We also observe that typically the rating of un-monitored substations is sufficiently large, as shown in Figure 16. In this case, even if the uncertainty of the estimated load is relatively high, this is not an issue, since the capacity of the substations is high enough to cover the maximum loading. This is demonstrated in Figure 16.

Figure 16 Comparison between substations’ estimated load during a peak loading condition and the substation rating

The estimated active and reactive power flows and the error margins for the peak loading condition are presented in Figure 17.

The sudden drop in the graph is due to the discontinuity caused by branch Joint-Substation 90625. The relative uncertainty of active and reactive power flows is depicted in the Figure 18. The level of uncertainty of the flow is quite small for the first 4 branches and the last 2, since data from the measurements is relatively accurate. The uncertainty of the flows in the branch Joint-Substation 90625 is quite high and close to the uncertainty of pseudo measurement, due to unknown loading of substation 90625. However, although the level of error of pseudo measurement data for substations 94356 and 91143 is 50%, the level of estimation error for the active power flows does not exceed 10% and for reactive power does not exceed 25%.

Figure 19 shows the comparison between the estimated maximum power flows and the circuit’s rating, for each section along the feeder BRXB-SE3. We observed that there is adequate capacity in this system as the maximum loading of the circuits is below 50%, even after taking into account the possible error in the estimation. Therefore we can conclude that as long as the capacity of the circuit can cope with the forecasted maximum loading of the circuit taking into account its maximum uncertainty in the estimation, there is no need to place an additional measurement for the respective circuit. However, if the maximum loading of the circuit may exceed the rating then it will be necessary to install an additional meter to have more accurate visibility. The results of year round study can be found in Appendix B.1.
Figure 17 Estimates of active and reactive power flow for the peak loading condition

Figure 18 Estimates of active and reactive power flow uncertainty for the peak loading condition

Figure 19 Comparison between the estimated power flows during the peak loading condition and the circuit’s rating, (BRXB-SE3)
From these analyses, it is concluded that the DSSE model is applicable and can provide accurate enough estimation of the system states and improve the accuracy of the measurements, whilst the application of pseudo-measurements can contribute to increased observability and accuracy of the estimation. It is noted that the uncertainty of estimated power flows is relatively high, particularly at the lateral sections which do not have measurements. However, this is already an improvement in comparison to having no visibility at all and the results can be improved if appropriate power flow measurement can be installed at the lateral sections in question. Given than the rating of the circuits is larger than the maximum flows (after taking into account its uncertainty), the error is not material. It is recommended to put power flow measurements at the sections of the network which are heavily loaded and need monitoring. For some circuits where the network loading is relatively low, the use of pseudo-measurement to improve visibility may be sufficient.

### 3.2.2 Year-round study to test the robustness and performance of the DSSE model

In this section, we describe the results of our analysis in comparing the voltage data from the measurement and the estimated voltages as results from the DSSE model across one year period. The period of observation is from 01 March 2013 to 01 March 2014.

Half-hourly voltages measurements, and estimates across one year samples of data, and their differences for the primary node are presented in Figure 20. It is demonstrated that the differences between data from the measurement and the output from the DSSE model are between ±1% for about 94% of the cases. The remaining cases have a higher deviation (up to 4.8%). It should be noted that the voltages used in analysis were measured at the transformer T4 of the Brixton primary substation.

![Figure 20 Annual voltages (left diagram) and differences between estimation and measurement (right diagram) at the primary substation](image)

Figure 20 shows the recorded and estimated voltages, and their differences for one of the distribution transformers, i.e. distribution transformer 90044. It is demonstrated that in many of the periods, the difference between the estimated and recorded values is very small. The accuracy of voltage measurements is greater at the distribution substation than at the primary substations as voltage transformers, which introduce additional error in measurement, are not used. In more than 90% cases, for all distribution transformers equipped with measurements, the mismatch between the recorded and results of DSSE model is relatively small and not higher than the level of errors of the measurements,
which are respectively 1% for the primary node and 0.6% for the secondary substations. The errors can be reduced if voltage measurements, in addition to the existing current measurements, can be installed at the primary substations.

![Figure 21](image1)

**Figure 21** Annual voltages (left diagram) and the differences between estimated and measured values (right diagram) at distribution transformer 90044

Figure 22 shows the measured and estimated voltages, and their differences for distribution transformers 90862. In this case the differences show the greater variation and this might be due to greater dependency of the measured voltage to the loading of the distribution transformer.

![Figure 22](image2)

**Figure 22** Annual voltages (left diagram) and the differences between estimated and measured values (right diagram) at distribution transformer 90862

Figure 23 shows the measured and estimated voltages, and their differences for the distribution transformers 94192. It can be seen that for the majority of the half-hourly period differences are at about -0.5%. This should be further investigated in cooperation with UK Power Networks. For example the parameters of distribution transformer, i.e. impedances, transformer ratio, tap-position, might differ from the typical values assumed in this study.

![Figure 23](image3)
If the voltage measurements at substation 94192 are increased by 0.5%, the mismatch between estimates and adjusted measurements is less than 0.1% in more than 90% of the cases, as presented in Figure 24. However this correction does not noticeably affect the values of estimates of any of the feeder’s nodes. This is due to high level of instrumentation of this feeder with sensors.

From these analyses we can conclude that even though the majority of half-hourly period estimations are relatively good, there is a considerable number of cases where the difference between measurements and state estimation is rather high, reaching almost 12% in the case of substation 91045. This suggests that the accuracy of some of the recorded data may not be as good as specified in their technical parameters. The examples of bad data detection are demonstrated in the following section.

### 3.3 Bad data detection

#### 3.3.1 Error in voltage measurement

This section shows an example where the high differences between the recorded and estimated voltage are indicated by the DSSE model. Figure 25 shows the voltage estimates at substations 90069 and 90638 on the 14 March 2013. Throughout the day, the values from the measurement are consistently within the ± 0.6% around the estimates, except for two half-hourly periods, i.e. the voltages at 11.30 and
Comparing voltage measurements (shown in blue) in nearby periods with the voltage measurements at these two half-hourly periods they appear as expected. There is a similar trend for other substations, apart from the voltage measurements at substation 91045, where the values drop to 0.85 and 0.89 p.u., respectively, as shown in Figure 26. The active and reactive power measurements are as expected, indicating that the error is likely in the recorded voltage measurements.

The estimated feeder voltages for the half-hourly period in 14 March 2013 at 11:30 are shown in Figure 27. This demonstrates that one bad measurement data can affect the accuracy of the voltage estimation: the difference between the estimated and measured voltages exceeds the level of DSSE error margin.
If these erroneous measurements are excluded, the voltage estimates improves as shown in Figure 28 and Figure 29.

Figure 28 Estimated and measured voltages for substations 90069 and 90638 on the 14/03/2013 after excluding substation 91045 bad voltage measurement data

Figure 29 Estimated and measured voltages for substation 91045 on the 14/03/2013 after excluding substation 91045 bad voltage measurement data
The estimated feeder voltages, without the erroneous measurement, at 11.30 are shown in Figure 30. It demonstrates that the results of the DSSE model are now meaningful and data of all used measurements fit within the expected measurement boundaries around the estimated values.

![Figure 30 Estimated and measured voltages at the feeder BRXB-SE3 for the operating snapshot on 14 March 2013 at 11:30, after excluding the bad measurement data from substation 91045](image)

This means that the identified erroneous measurements have to be excluded from the final run of the DSSE model in order to achieve the estimation of satisfactory quality.

### 3.3.2 Enhancing the accuracy of pseudo measurements using typical load profiles

The following examples demonstrate the importance of the pseudo measurements. Due to a malfunction of the system on the 16 September 2013, the recording of load measurements in substation 94192 failed to work from 10.00, followed by failed recording in substation 90043 at 12.00 and in substation 91045 at 13.00, until the following day. The recordings of measurements for the rest of the substations were not interrupted. For the purpose of estimation, the gap in the real measurements has to be filled with pseudo measurements.

There are two approaches investigated in this study. Approach 1: if the typical load profiles of the substations are unknown, the best approximation of the substations loadings can be calculated as the difference between the known loadings of the feeder and the known loadings of the substations and distributed among the substations without measurements based on their capacity. In this particular study, the load difference is distributed equally among all unmonitored substations, including the ones which are temporarily unmonitored due to failure in their data recording. The second approach (Approach 2) uses the typical load profiles of the substation as the basis for distributing the load obtained by subtracting the load at primary substation with the recorded load from secondary substations.

The estimated active power flows for both cases are compared and the results are presented in Figure 31, Figure 32 and Figure 33, for the substations 94192, 90043, and 91045, respectively. The left and
right diagrams represent the results of the first and the second approach respectively. The blue colour line denotes the measurement data and the red the output from the DSSE model.

Figure 31 The estimated and the measured active power flows for substation 94192 by using Approach 1 (left diagram) and Approach 2 (right diagram) pseudo measurement approach

Figure 32 The estimated and measured active power flows for substation 90043 by using Approach 1 (left diagram) and Approach 2 (right diagram) pseudo measurement approach

Figure 33 The estimated and measured active power flows for substation 91045 by using Approach 1 (left diagram) and Approach 2 (right diagram) pseudo measurement approach

In the periods when data from real measurements are available, the estimate values are similar to the recorded data with relatively small differences. At the moment of loss of measurements the uncertainty of the estimate values increase up to 50%. As shown in the left diagram of Figure 31, we observe that, in
the moment of loss of the measurement data, there is no smooth transition from previously recorded values. The values of active power as one would expect, do not fit within the accuracy boundaries of estimates for distribution transformers 94192 and 90043. However, by using the information from the typical load profile to calculate the pseudo measurements, this transition is smooth and gives more meaningful average estimates even though the uncertainty is still high.

3.4 Sensitivity analysis

3.4.1 Impact of voltage measurement accuracy

In this section, the impact of the accuracy of voltage measurements is investigated. We consider two different levels of accuracy; at primary site we use 0.1% and 1% and at secondary site we consider 0.3% and 0.6%.

The results of DSSE model for the measurements accuracy of 1% and 0.3% for primary and secondary substations, respectively for the peak loading and minimum loading conditions are presented in Figure 34. It can be observed that the voltage measurement of substation 94192 do not fit in the ±0.3% boundaries around estimates. However, if the measurement is corrected for a bias as described in section 3.2.2 it will fit in.

![Figure 34 BRXB-SE3 estimated and measured voltages for max and min loading conditions assuming error in measurement at primary substation is 1% and at secondary substations 0.3%.](image)

Figure 35 presents probability distribution of mismatches between the measured and estimated voltages for three of monitored substations, during the year. For two of those secondary substations the mismatches are distributed around zero while for substation 94192, the mismatches are distributed around 0.5%, as already presented in section 3.2.2. It can be seen that the majority of differences can fit in range of 0.2% even for the substation where bias of 0.5% is detected. As pointed earlier that the reason for bias can be investigated and measurements corrected for bias.
Further sensitivity analysis considers impact if accuracy of secondary substation voltage measurements is worse, 0.6% instead of 0.3%. Figure 36 presents the voltage estimates for the peak and low demand conditions. It can be noticed that the new results better fit the majority of the measurements, although voltage estimation uncertainty is greater than the ones shown in Figure 34.

Figure 36 demonstrates the estimated voltages for peak and minimum loading conditions, in the case of high accuracy of primary node measurements, with the error of 0.1% and secondary substation voltage measurement accuracy of 0.6%. Given higher accuracy of the primary node, voltage estimations are shifted up towards the measured voltage at the primary substation.
From the year-round studies, the mean value of differences between the measured and estimated voltage are compared if level of error of the voltage measurements at primary substations is 0.1% and 1% and shown in Figure 38. If the error is 1% the mean value is close to 0 for all substations except substation 94192, whose mean value is 0.5% as already presented earlier. However, if the primary node voltage measurement error is equal to 0.1%, the mean value of voltage difference at primary node, as expected, is closer to zero. However, the average differences for voltage at the secondary substations are considerably higher.

Based on the discussions above, the study demonstrates that overestimating the accuracy of the meters will lead to inaccurate system state estimation.

### 3.4.2 Impact of accuracy in network parameter data on the accuracy of DSSE output

In this section the robustness of the results of the DSSE model against the inaccuracy in determining the circuit parameters, such as resistance and reactance, is analysed. In this case, the impedance is increased by 50%.
Figure 39 compares the probability distribution of mismatches between the estimated and the measured voltages taking into account all measurement samples for both cases, i.e. the base-case and the case with increased impedance. The comparison is carried out for both cases for two substations 90069 and 94192. The results suggest that the probability of errors for substation 90069 increases in the 2nd case while the opposite trend is observed for substation 94192. One may conclude that the network data until substation 90069 which is located close to the primary substation is relatively accurate while the network impedance seen from substation 94192 to the primary substation should be higher than what has been recorded. It should be noted that the resistivity of the circuits changes with the conductor temperature. Thus, there is uncertainty around the network parameters that should be taken into account in state estimation. This will require further investigation and development of more advance DSSE algorithms which are specially designed for distribution network applications.

![Figure 39](image.png)

**Figure 39 Comparison of differences between measured and estimated voltages and accuracy boundaries for all periods between the case of increased impedance and the base case for BRXB-SE3 feeder**

For the implementation of DSSE on distribution networks, it is therefore important to establish the database of network parameters with sufficient accuracy. The level of accuracy required needs to be investigated and agreed with network planners and operators before the implementation of DSSE. Furthermore the importance of the level of accuracy will be case dependant and the materiality will be driven by the headroom.

### 3.4.3 Impact of sensors availability to SE

**Studies on an operating snapshot**

In order to evaluate the accuracy of the SE, a set of studies with a smaller number of measurement points has been carried out. The estimated voltages and the estimated active power flows are compared with the “true” values obtained by running the DSSE model taking into account all available measurement data. The “true” values are used as reference values. The comparison between the estimates and the reference values of voltages and active power for various measurement configurations is presented in Figure 40 and Figure 41 respectively. The comparison uses only a snapshot of network operating condition on 31 January 2014 at 23:30.
If only measurement data from the first substation at the beginning of the feeder are taken into account, the estimated voltage (V estimate M1) for this substation is close to its recorded voltage, but the results deviate from the reference values at all other nodes, as shown by the blue line. If the measurement data from the first node and the last node (V estimate M1&12) are taken in account, the deviation from the reference values decreases as shown by a solid purple line in Figure 40. Adding a flow measurement at a line between node 8 and 9 (please see Table 2 to find the respective substation id) improves slightly the results, depicted by orange colour.

It is interesting to observe that additional measurements will not always improve the estimation. For example, the results of the study using measurement data from node 1, 11 and 12, as presented with green colour in Figure 40, show further deviation to the reference values in comparison with the results of using measurement data from node 1 and 12. This is similar with the case where measurement data from node 1, 7, and 12 are used. This increased deviation is caused by the characteristic of measurements at node 7 and 11. As shown in Figure 40, the measured voltage at node 7 is higher compared to the reference value, thus when we consider to include this measurement in the DSSE calculation, the estimated voltages of the system tend to increase. Similarly, the measured voltage at node 11 is down below the reference value and therefore, by including this measurement in the DSSE calculation will pull down the estimated voltages.

![Figure 40 Comparison of estimated and true voltage in the case of availability of sensors on various locations](image)
The comparison between the estimated and reference values of active power flows for this study are presented in Figure 41. In general, we observe a similar trend as being observed in the voltage study above. The state of the system can be estimated only by using few measurements and additional measurement data, on the one hand, will generally improve the quality of the estimation. On the other hand, inaccurate measurement data reduce the quality of estimation, thus the accuracy of the measurement is the key factor in the implementation of SE.

It is important to note that the implementation of DSSE model will improve the accuracy of the monitoring system subject to sufficient availability of real measurements with sufficient accuracy.

**Studies on a large set of measurement data samples**

In order to further evaluate the impacts of having different number of measurements at different locations, a set of studies has been carried out by excluding the measurement data from 1 up to 5 substations those have real measurements. It may be important to mention that substations at node 6, 8 and 10 do not have measurements. The studies were carried out evaluating different network operating conditions occurred in 2 weeks from 18 to 31 January 2014. The estimated values in cases with fewer sensors are compared with the reference values.

There are 6 cases:
- Case no 1: This study examines impact of excluding the measurement data from 1 substation (only one at a time) at node: 2, 3, 4, 7, 9, 11, and 12.
- Case no 2: This study examines impact of excluding 2 measurement data from the selected locations: 2 and 3, 3 and 4, 4 and 7, 7 and 9, 9 and 11, and 11 and 12.
- Cases no 3-6: This study examines impact of excluding 3 to 6 measurement data from the selected combinations of secondary substations, respectively.
The key results of the studies are discussed as follows.

First, we analyse the impacts on estimated voltages. Then we calculate the average deviation of the estimated voltages from the reference voltages at each node across all samples for each scenario. As Case 1-6 involve more than one scenario (the condition where we exclude the measurements at specific locations), we identify the minimum and maximum average deviation in each case and the respective drivers. The results are showed in Figure 42, and the drivers of the results are listed in Table 5.

The results as shown in Figure 42 demonstrate that having fewer measurements will lead to more inaccurate results. We observe the increased mean of error (deviation between the estimated and reference values) in the cases with fewer measurements available. The deviation can be in both directions (positive or negative) thus it is difficult to predict whether it is over or underestimated. The impact of having fewer measurements is also non-linear and likely to be case specific.

Table 5 shows the locations of the measurements which are excluded from the DSSE calculation that lead to the results shown in Figure 42. For example, excluding measurements at node 9 or node 11 in Case 1, will have the maximum impact on the DSSE if the system should lose one measurement. This implies the sensitivity of the DSSE output with respect to the measurement data from these locations.
Table 5 The drivers of the results illustrated in Figure 42

<table>
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<tr>
<th>Index</th>
<th>Node</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
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<th>Case 5</th>
<th>Case 6</th>
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From this exercise, one can also identify the measurements that have relatively significant influence on the output of the DSSE model. From the table above, measurements at the end of feeders such as at node 11 and 12 have been identified in many cases as important drivers for the accuracy of the SE.
4 Applications of DSSE Model for other EIZ Feeders

4.1 Application of DSSE for Feeder MERT-E2 from Merton EIZ

HV feeder MERT-E2 belongs to Merton EIZ. Its total length is 7.29 km and supplies around 3718 domestic, industrial and commercial customers via fourteen 11kV/0.4kV secondary substations. Two substations (1st and 8th) have capacity of 0.75 MVA and 0.8 MVA respectively and the other substations have capacity of 0.5 MVA. The diagram of the feeder is given in Figure 43.

In this specific example, the majority of the distribution transformers are instrumented with RTUs. Red and orange ellipses mark the substations where some or all measurement data are not available or not of good quality. For example, there is no data for substation 6635 in the ODS database. The orange ellipse marks substation 8462, where only records of phase voltages are available but not active and reactive power due to the limitation in installing a new current transformer on this site. The dotted red ellipses mark the substations equipped with the new type of RTU. Unfortunately, the measurements from these new RTUs are infrequent and erroneous. Substation 6248 measurements appear in 4 months but the data shows negative consumption, although no generation was connected. Other new RTUs’ data was quite irregular. The blue dotted ellipse marks the primary substation node with only

![Figure 43 Feeder MERT-E2, a) ENMAC, b) simplified diagram](image-url)
current measurements. By using the current measurement data and the voltage measured at the closest node, we derive the active, reactive power flows at the primary node.

The list of the nodes with the capacity of substations, the total and numbers of customers of different classes is presented in Table 6. Substations with no measurements are marked red. Network data are presented in Table 7.

Table 6 Data from substation MERT-E2. There are no measurements at nodes marked red, substation 8462 has only voltage measurement and the first node is marked blue

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Table 7 MERT-E2 Feeder network data

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<th>to node</th>
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<th>Branch Constr.</th>
<th>Resistance (R) (% at 100 MVA base)</th>
<th>Reactance (X) (% at 100MVA Sbase)</th>
<th>Line Rating - min(A)</th>
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4.1.1 Results on the application of DSSE for Feeder MERT-E2 from Merton EIZ

In this section, we describe the results of our analysis in comparing the estimated voltage obtained from the DSSE model and the measured voltages across one year period (from 01 March 2013 to 01 March 2014). We assume the error margin of the voltage measurements in the primary substation is 1.5%, and in the secondary substation is 0.6%. Half-hourly measurement data, the estimates, and their differences for a number of nodes are presented in Figure 44 - Figure 46.

Figure 44 The estimated and measured voltages (left diagram) and their differences (right diagram) for nodes: FDR019, 8150 and 8151
Figure 45 The Estimated and measured voltages (left diagram) and their differences (right diagram) for nodes: 6601, 8507, and 8462
Figure 46 The estimated and measured voltages (left diagram) and their differences (right diagram) for nodes: 8050, 6243, and 6552

In general, the estimated voltages are relatively accurate; however there are some instances where the error is relatively large. This will require further investigation in the measurement data to establish their accuracy and whether there was a change in the system state that had not been recorded properly.

We also observe in Figure 46 that at some nodes including node 6243, in most of the times the differences between the estimated and measured voltages are not zero, this implies that the measurements may require some corrections. In the following example, we apply 0.7% correction to the voltage measurement data from substation 6243. Figure 47 shows a better graph demonstrating that in most cases the deviation of the estimated voltages from the measured voltages is close to zero.
In Figure 48, we demonstrate that after the correction, all measured voltages are now within the margin of the error of the measurement devices assuming that the state of the system is the one depicted by the DSSE model.

Figure 49 demonstrates the uncertainty of voltage estimation based on the measurement data of the operating snapshot on 24 February 2014 at 18:30. The uncertainty is relatively small (approx. 0.2%). The trend shows that the farther the substation from the primary, the uncertainty increases.
The uncertainty of voltage estimation based on the operating snapshot on 24/02/2014 at 18:30

The left and right diagram of Figure 50 show the comparison between the estimated values and measured values of active and reactive power respectively for the peak demand condition. The results demonstrate the measured values are within the error margin of the estimated values.

The uncertainty of estimated active power is shown in Figure 51. The uncertainty is relatively low, less than 6-7%, for the substations that have measurements but is high, more than 50%, for the substations that have no respective measurements (circled red in Figure 51). The results are driven by the use of pseudo measurements to substitute the missing measurement. The results do not suggest that the uncertainty increases for the substation farther from the primary substation; this is different compared to the uncertainty in the estimated voltage that has been discussed previously. The results of the full year round study can be found in Appendix B.4.
Figure 51 The uncertainty of estimated active power based on operating snapshot on 24/02/2014 at 18:30

Figure 52 Estimated active power flows and its uncertainty based on operating snapshot on 24/02/2014 at 18:30

Figure 52 (left diagram) demonstrates the estimated active power flows for all branches. As expected, the flows decrease along the feeder. Figure 52 (right diagram) shows the uncertainty in the estimated active power flows. Consistent with the results for active power estimation, the uncertainty of estimated active power flows at the lateral branches is relatively high due to the use of pseudo measurement (with 50% error margin) to substitute the missing measurement data. This implies that for these lateral branches, more accurate estimation can only be obtained by putting power flow measurements in place.
## 4.2 Application of DSSE model for Feeder AMBL-NW1 from Queen’s Park EIZ

AMBL NW1 feeder belongs to Queen’s Park EIZ. Its total length is 1.9 km and it supplies around 1520 domestic, industrial and commercial customers via four 0.8 MVA rated 11 kV/0.4 kV transformers. The graphical interpretation of the feeder is given in the Figure 53.

Red ellipses mark the substations with no measurements. The primary readings are completely absent from ODS and there is no RTU installed on substation 30166.

![Graphical interpretation of the feeder](image)

**Figure 53 Feeder AMBL NW1 – Queens Park, a) ENMAC, b) simplified diagram**

The list of the substations with their capacity and the number of customers for various classes is presented in Table 8. The explanation of the customer class profiles can be found in the Appendix. The substations with no measurement data are marked red. The network data used for this study can be found in Table 9 below.

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<td>61</td>
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Table 9 AMBL-NW1 Feeder network data

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<th>From node</th>
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<th>Resistance (R) (% at 100MVA)</th>
<th>Reactance (X) (% at 100MVA)</th>
<th>Line Rating –min (A)</th>
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### 4.2.1 Results on the application of DSSE for AMBL-NW1 feeder

Since there is no measurement data obtained from substation 30166 or from the primary, in order to perform the state estimation we use pseudo measurements. We assume that the load of substation 30166 is proportional to the load of other 3 substations. Then, the load of the primary substation is calculated as a total load from all substations. The error margin of the pseudo measurements is assumed to be 50%. Taking into account there are no measurements for the primary voltage either, the nominal voltage of 11 kV is assumed, with a large error margin, i.e. 6%. The error margin of the known substation voltage measurements is considered to be 0.3%. The study excludes the periods (around 300 half-hourly snapshots) where there are no data available.

![Node FDR004 Estimated and Measured Voltages](image1)

![Node FDR004 Differences](image2)

![Node 30027 Estimated and Measured Voltages](image3)

![Node 30027 Differences](image4)

Figure 54 The estimated and measured voltages (left diagram) and their differences (right diagram) for the primary substation and the distribution transformer 30027

55
Figure 54 and Figure 55 show voltage estimation for a year round analysis. Charts on the left hand side show chronological estimated and measured voltages while charts on the right hand side show the sorted differences between average estimated and measured voltages. It can be seen that the average differences are quite small for all distribution substations.

In most of the cases, the errors are relatively small except the errors of the estimated voltages for the primary substation. These errors are relatively large driven by the uncertainty in the accuracy of pseudo measurements that have to be used to fill the gap in the measurement data.

Figure 56 shows the comparison between the estimated and measured voltages for the peak demand condition recorded on 24 March 2013 at 19:30. The results are very encouraging as the mismatches
between the estimated and measured voltages are relatively small and well below the error margin of the estimation.

Figure 56: The estimated and measured voltages for the peak loading condition

Figure 57 demonstrates that the uncertainty of voltage estimation based on the measurement data of the operating snapshot on 24/02/2014 at 18:30. The uncertainty is relatively small (approx. 0.17%). The trend shows, consistently with the previous study, that the farther the substation from the primary, the uncertainty increases.

The left and right diagram of Figure 58 show the comparison between the estimated values and measured values of active and reactive power respectively. The results demonstrate the measured values are within the error margin of the estimated values.
The uncertainty of estimated active power is shown in Figure 59. The uncertainty is below 7%. The results do not suggest that the uncertainty increases for the substation farther from the primary, which is consistent with the results of the previous study. In this case, the uncertainty of active power estimation for the substation (e.g. 30166) that has no measurements (circled red) is also higher than other substation. However, in this case, the uncertainty is kept below 7%.

Figure 60 and Figure 61 (left diagram) demonstrate the estimated active and reactive power flows for all branches respectively. As expected, the flows decrease along the feeder.
Figure 60 Estimated active power flows and its uncertainty evaluated based on the peak loading condition

Figure 61 Estimated reactive power flows and its uncertainty evaluated based on the peak loading condition

Figure 60 and Figure 61 (right diagram) show the uncertainty in the estimated active and reactive power flows respectively. Consistent with the results for active and reactive power flows estimation, the uncertainty of estimated active and reactive power flows is relatively small and kept below 2.5% for active power flows and 3% for reactive power flows. The results of year round study can be found in Appendix B.3.
4.3 Application of state estimation for the Feeder BRXB-NE2 from Brixton EIZ

HV feeder BRXB-NE2 belongs to Brixton EIZ. It has seven 11kV/0.4kV secondary substations as shown in Figure 62. Four distribution substations are instrumented with RTUs. Red ellipses mark the location of substations where some of measurement data are missing or not of a good quality. There is no RTU installed on substation 90291 and 90081. The dotted red ellipse marks substation 90153 with 2 transformers T1 and T2. There are customers supplied from T1 but no any measurements available. However, there is no information about customers supplied from T2 but there are voltage measurement data, which are excluded from the DSSE since they are considered erroneous (information from UKPN). The blue dotted ellipse marks the primary substation node of the feeder. It has only a current measurement. By using the current measurement data and the voltage measured at the closest node, we derive the active, reactive power flows at the primary node.

![Diagram](image)

The substation data and network data are presented in Table 10 and Table 11 respectively.

**Table 10 Data from substation BRXB-NE2.** There are no measurements at nodes marked red, and primary node is marked blue.

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Table 11 BRXB-NE2 Feeder branches data

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<th>to node</th>
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<th>Branch Constr.</th>
<th>Resistance (R) (% at 100MVA)</th>
<th>Reactance (X) (% at 100MVA)</th>
<th>Line Rating –min (A)</th>
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<td>90415</td>
<td>90081</td>
<td>623.08</td>
<td>185</td>
<td>7.33</td>
<td>4.12</td>
<td>280</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>6</td>
<td>90081</td>
<td>90621</td>
<td>341.02</td>
<td>185</td>
<td>4.30</td>
<td>2.18</td>
<td>280</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>7</td>
<td>90621</td>
<td>90153</td>
<td>709.75</td>
<td>0.15</td>
<td>10.90</td>
<td>4.65</td>
<td>240</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>7</td>
<td>90282</td>
<td>90153</td>
<td>290.99</td>
<td>0.15</td>
<td>4.96</td>
<td>1.94</td>
<td>240</td>
</tr>
</tbody>
</table>

4.3.1 Results on the application of DSSE for feeder BRXB-NE2 from Brixton EIZ

The results of the study are presented in Figure 63 and Figure 64. The left diagram of those figures presents the estimated and measured voltages across all snapshots under the study and the right diagrams show the differences between the two values presented in an increasing order.

![Figure 63 The estimated and measured voltages (left diagram) and their differences (right diagram) for the primary substation and substation 90416](image)
Figure 64 The estimated and measured voltages (left diagram) and their differences (right diagram) for substation 90416, 90415, 90621, and 90282

The results demonstrate that the average differences between the estimated and measured data are close to zero in most of the cases while there are some instances where the errors are relatively high. The results look consistent with other previous results, and one can conclude that the DSSE approach used in the DSSE model is relatively robust and suitable for real applications on distribution networks.

Figure 65 shows the estimated voltages along the feeder evaluated based on the peak demand condition. In this case, the measured voltages at the primary, substation 9012, and substation 90282 are a bit far off from the estimated values but still are within the margin error of the measurements.
Figure 65 Estimates of Voltages for the peak loading condition

Figure 66 shows the uncertainty of estimated voltages. The results are encouraging as the error margins are relatively low, i.e. around 0.15%.

Figure 67 shows the comparison between the estimated active (left diagram) and reactive (right diagram) power demand for the peak loading condition.

Consistent with the previous results, we observe that the differences between the estimated values and the measurement data are relatively small at the substations that have power measurements. The uncertainty is higher for substations of which pseudo-measurements are used. This is also demonstrated in Figure 68 where the uncertainty of estimated active power demand estimation is presented.
Figure 67 The estimated active and reactive power evaluated based on the peak loading condition

Figure 68 The uncertainty of estimated active power evaluated based on the peak loading condition

Figure 69 and Figure 70 (left diagram) demonstrate the estimated active and reactive power flows for all branches respectively. As expected, the flows decrease along the feeder. As expected, more accurate estimation is obtained due to the availability of the real measurements at the start and at the end of feeder. In the middle of the feeder the accuracy is lower due to lack of measurements at some of the distribution substations.
Figure 69 and Figure 70 (right diagram) show the uncertainty in the estimated active and reactive power flows respectively. Consistent with the results for active and reactive power flows estimation, the uncertainty of estimated active and reactive power flows is relatively high and reaches 22% for active power flows and is close to 24% for reactive power flows. The results of year round study can be found in Appendix B.2.

The results of our studies for four EIZ feeders are compared and summarised in Table C.1 and Table C.2 in Appendix C: Uncertainty in Estimated Voltages and Power Flows. Our studies suggest that the uncertainty (error margin) of the estimated voltages is relatively small and in most of the cases, the error margin is less than 0.22% (in comparison to the error margin of individual meters: 0.3%-0.6%). We also observe that by having the right measurements at substations, the error margin of the estimated power flow can be reduced to typically below 10%. The use of state estimation can also enhance network visibility for the lateral sections where the substations have no measurements installed.
5 Studies of Meter Placement along EIZ Feeders

Due to vast number of nodes in a distribution network, it is neither practical nor economically justified to have a fully instrumented network. State estimation can be used to reduce the number of meters needed but strategically located to ensure that adequate network observability and the accuracy of state estimation can be achieved over a range of different operating scenarios.

The aim of this work is to identify the number, location and type of measuring equipment to achieve a desired state estimation quality. In this work, we aim to have the uncertainty of estimated voltages and power flows below 1% and 10% respectively. The criteria are selected taking into account the statutory operating limits of voltage (±6%) and the fact that most of the branches have thermal capacity double than the maximum load recorded in the ODS database. Standards on the state estimation quality may need to be developed for distribution network applications.

The objective of developing the meter placement methodology is to reduce the relative errors in voltage and angle estimates for the whole feeder, below pre-specified thresholds in more than 95% cases of a representative set of different operating snapshots as measured and recorded in the ODS database. To achieve diversity in operation scenarios, the probabilistic method based on Monte Carlo simulations is used.

5.1 Methodology for recommended sensor placement strategy

The methodology for recommended sensor placement strategy employs the idea of sequentially improving a bivariate probability index, based on the relative estimation errors in voltage and angle at each substation with the help of the two-sided Chebyshev inequality [5].

The problem of meter placement is to identify the effective locations and the number of measurements, so that the probability indices ($p_i$), for each node $i$, related to the relative errors of the voltage and angle estimates are improved and relative errors are brought below their specified thresholds. The probability indices for each node $i$ are defined as:

$$ p_i = \text{Probability}\left\{\frac{\hat{V}_i - V_t^i}{V_t^i}, \frac{\delta_i - \delta_t^i}{\delta_t^i} \leq \epsilon_1, \epsilon_2\right\} $$

(4)

where $\hat{V}_i$, $\delta_i$ - Estimated value of voltage and angle at the $i$th node, respectively;

$V_t^i$, $\delta_t^i$ – “True” value of voltage and angle at the $i$th node, $i=1, ..., n$. The “true” voltages are the voltages obtained by state estimation on fully instrumented network, and the “true” values of the voltage angles are the calculated values of angles obtained from power flow analysis on the fully instrumented network. It is important to note that the reference values shall not be obtained from individual physical measurements as they may be inaccurate/ erroneous, it is more reliable to use the results of DSSE model which consider all available information as the reference values;

$\epsilon_1$, $\epsilon_2$ - specified relative error thresholds for voltage and angle.

The solution of this problem is based on the reduction in the Chebyshev bound, which is achieved by reducing the area of the mean error ellipse generated by the error covariance matrix $P_{x_i}$ for each node $i$ of the feeder over the whole set of Monte Carlo simulations. As the error ellipse is proportional to $\sqrt{\det P_x}$, the meters are introduced at the locations where this determinant is the largest.
5.1.1 Meter placement algorithm

The algorithm to decide the location and the type of new meters is the following:

**Step 1** Perform DSSE over a set of scenarios and observe the relative errors in voltages and angles at all substations in each simulation, see Figure 71.

**Step 2** If the relative errors in the voltages and angles are below their specified thresholds respectively (e.g. 1% for voltage and 10% for angle, indicated by the red line on the graph) in more than 95% of the cases, no additional meters are required, therefore **Stop**, if only angle estimates violate the threshold, go to **Step 4**, otherwise continue to **Step 3**.

**Step 3** Take the mean of the state error covariance matrix over all the Monte Carlo simulations and extract the sub-matrices corresponding to the voltage and angle at each node. Compute the area of the error ellipse at every bus from the determinant of its error covariance matrix. Identify the node with the largest error ellipse area and place appropriate measurements at this node. If the measurement is already present, choose the node with the next largest area. Go to **Step 1**.

**Figure 71** Relative errors in voltage and angle estimate with initial number of metering location

Figure 72 shows examples of error ellipses for some of feeder nodes. The x axis denotes error in angle while y axis denotes error in voltage. A red ellipse presents a situation where the number of meters in the network is low (before new meters are installed) and a blue ellipse represents the situation where a set of additional meters has been put in place. The key point here is to place additional meter at the node with the largest error ellipse. As illustrated in Figure 73, placing meter on almost any location can contribute to the reduction in the error ellipse of the feeder nodes, however, this location should be such to reduce it in the best way for the particular feeder. The blue ellipse obtained by some random location will indeed reduce error ellipse, but in this particular case it will reduce only the error of the
angle for this particular node, however, the voltage error will stay the same. For other nodes it could have the opposite effect.

Figure 72 Error ellipse examples

Figure 73 Error ellipse example

Step 4 Placement of Line Power Flow Measurements - compute the mean of error covariance matrix corresponding to the real and reactive power flow in each line. For each line compute the area of the line flow error ellipse and place the flow measurement in the line with the largest area. If the measurement is already present, choose the line with the next largest area. Go to Step 1.

5.2 Sensor placement analysis – Brixton feeder BRXB-SE3

In this exercise, the measurement data for a two-week period at the end of January 2014 is used which provides 672 different operational scenarios. Moreover this period is selected as the differences

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5 This example is based on the meter placement studies on the Rossenhallas EDF 33/11 kV feeder in the previous work [6]
between the measurement data and estimated values are low. It is assumed that the measurements on the beggin of the feeder at the primary substation are always available. We start the analysis assuming there are no measurements installed at any of secondary substations. This implies the use of pseudo measurements to model the secondary substation loads. The value of pseudo measurements is recalculated when there is new available load-measurement data from secondary substations. In order to introduce more diversity in the possible operation scenarios, these pseudo measurements are further adjusted by adding random errors assuming the distribution of errors follows normal distribution.

The errors of the measurements are taken as defined earlier: 1% for the voltage of primary substations and 2% for the active and reactive power flows.

5.2.1 Case study: Base case - Measurement available only on the first feeder node

Step 1

The difference in voltage estimates and true voltage for the peak loading snapshot are presented in the Figure 74. Average estimated voltages, for each node, deviate from the true values around 0.3%. However, the estimated accuracy range is wide, at around ±1%, and the true values fall within the range.

One could argue that just with readings on the primary node we have achieved the sufficient accuracy with 1% level of error. However, it should be taken into account that this particular feeder is relatively short and lightly loaded (the peak current on the top of the feeder reached in examined year period is 131 A and the lowest rating of all branches is 280 A), the voltage drop from the beginning to the end of the feeder is relatively small, i.e. 0.6%. Even if each substation loading is increased 50%, the voltage drop would still be relatively small, i.e. 0.8%.

![Figure 74 Voltages estimates and true voltages for the peak loading snapshot. Base case study: Measurement available only on the first node](image)

Furthermore, we observe that the inaccuracy of primary node measurements will affect the accuracy of SE, as presented in Figure 75. In this figure, the comparison of voltages estimates and the true value for the substation 91045 for the 2 weeks periods is presented.
The active flow estimates are depicted in Figure 75 for the first and the last five branches. As expected, the farther from the supply point the higher is the difference between the estimates and the true values and the level of uncertainty ("error") increases due to the use of pseudo measurements. As presented in Figure 10, although having the same rating, typical loading of some substations can differ even threefold as in the case for substation 91045.
Figure 76 Estimated and true active power flows for the 2 weeks period. Base case study: Measurement available only on the first node

It can be concluded that having just one measurement on the supply point is not sufficient to provide adequate estimation of the network state.
Step 2

Relative errors in voltage and angle estimates are presented in Figure 77 and Figure 78, respectively. There are a considerable number of the cases where the relative errors in voltage estimation are greater than 1%. The relative errors in angle are relatively large.

![Figure 77 Relative errors in voltage estimate. Base case study: Measurement available only on the first node](image1)

To improve the estimation an additional measurement is needed and its location is determined in the next step.

Step 3

The area of the error ellipse is calculated for every node from the determinant of its error covariance matrix and presented in Table 12. In this case the error covariance matrix indicates how the errors in
voltage magnitude and voltage angle change together. The impact on the accuracy of the estimation will be the largest by placing an additional meter in the place where the errors are the greatest. The graphical interpretation of voltage and angle error ellipses for each node is presented in Figure 79. The X axis represents the variance in angle and the Y axis the variance in voltage. The substation 91045 node is identified as the node with the largest error ellipse area indicating the new measurement should be placed there.

Table 12 Determinant of error covariance matrix if measurement is only available at the primary for feeder BRXB-SE3

<table>
<thead>
<tr>
<th>Node</th>
<th>90069</th>
<th>90044</th>
<th>90043</th>
<th>Joint</th>
<th>90625</th>
<th>90862</th>
<th>94356</th>
<th>90638</th>
<th>91143</th>
<th>94192</th>
<th>91045</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{\det P_x}$</td>
<td>3.76E-09</td>
<td>1.48E-08</td>
<td>2.50E-08</td>
<td>2.76E-08</td>
<td>3.74E-08</td>
<td>7.16E-08</td>
<td>9.15E-08</td>
<td>1.23E-07</td>
<td>1.64E-07</td>
<td>1.65E-07</td>
<td></td>
</tr>
</tbody>
</table>

Figure 79 Error ellipses for each substation if measurement is only available at the primary substation

5.2.2 Case study: Additional measurement available on the last node

Step 1

As the outcome of the first iteration of meter placement algorithm, an additional meter has been placed at the last substation.

The differences between the estimated voltages and true voltages for the peak loading condition are presented in Figure 80. The differences are relatively small - less than 0.037% for each node, well within the error tolerance of the measurements, which is about 0.6%.
Comparison of voltages estimates and the true value for the substations 91045, where the additional measurements are placed and substation 90862 from the middle of the feeder are presented for the period of 2 weeks in Figure 81. The rest of the substation voltages follow the similar pattern. The voltage mismatch is generally small, only in few cases it is exceeding 0.5%.

The active flow estimates for the first and the last 5 branches are depicted in Figure 82. As expected, the mismatch between the estimated and the true flows is almost negligible for the first and the last branch, as the measurements are known, and slightly increasing towards the middle, but still within 3 standard deviations boundaries. These boundaries, and therefore uncertainty in estimates are the highest for the branch joint-90625, as this substation is off the main subsequent line of supply and the loading uncertainty is assumed to be 50%.
Figure 82 Estimated and true active power flows for the 2 week period. Case study: Measurement available on the first and the last node

Step 2

Relative errors in voltage and angle estimates for all 672 examined cases are presented in Figure 83 and Figure 84 respectively. The voltage estimation in the case with additional measurements on the last
node has been improved significantly. The relative errors in voltage exceed 0.5% only in 3 simulations. The relative errors in angle decrease but still relatively high.

Therefore the next additional meter should be considered. Since the voltage is within satisfactory boundaries, but angle is not, the sensor placement algorithm suggests placement of line power flow measurements.

**Step 4**

The area of the Flow error ellipse at every branch is presented in Table 13, and the graphical interpretation of active and reactive power error ellipses is presented in Figure 85. The Blue line presents the case when there are measurements only on the first node and the red when the additional meter is placed. Installing measurements at the last substation has improved significantly the power flow estimation. The comparison of \( \sqrt{\text{det}(\mathbf{P})} \) in cases of measurement availability on the first, and on the
first and the last node is presented in Table 13 and Figure 86. It can be seen that in both cases Flow error ellipse recommend placement of flow measurement in the branch 8-9.

![Figure 85 Comparison between the active power flow error ellipses for each branch in the case where measurements are available at the first node and in the case where the measurements are available at the first and the last node.](image)

**Table 13 \sqrt{\det(P_{ij})} Case study: Measurement available on the first and the last node**

<table>
<thead>
<tr>
<th>Branch</th>
<th>1-2</th>
<th>2-3</th>
<th>3-4</th>
<th>4-5</th>
<th>5-6</th>
<th>5-7</th>
<th>7-8</th>
<th>8-9</th>
<th>9-10</th>
<th>10-11</th>
<th>11-12</th>
</tr>
</thead>
<tbody>
<tr>
<td>\sqrt{\det(P_{ij})} M1</td>
<td>1.14E-09</td>
<td>1.05E-08</td>
<td>1.85E-08</td>
<td>2.53E-08</td>
<td>9.58E-09</td>
<td>3.08E-08</td>
<td>3.48E-08</td>
<td>3.81E-08</td>
<td>3.57E-08</td>
<td>3.19E-08</td>
<td>4.78E-09</td>
</tr>
<tr>
<td>\sqrt{\det(P_{ij})} M1&amp;12</td>
<td>1.14E-09</td>
<td>7.45E-09</td>
<td>1.29E-08</td>
<td>1.74E-08</td>
<td>6.55E-09</td>
<td>2.1E-08</td>
<td>2.36E-08</td>
<td>2.5E-08</td>
<td>2.3E-08</td>
<td>2.01E-08</td>
<td>2.38E-11</td>
</tr>
</tbody>
</table>

![Figure 86 comparison of \sqrt{\det(P_{ij})} in cases of measurement availability on the first and on the first and the last node.](image)
5.2.3 Case study: Measurements available on the first, last node and flow at the branch 8-9

For the purpose of this exercise, the measured flow in the branch 8-9 is assumed to be the average estimated flow when all available measurements are considered.

Relative errors in estimated voltage and angle are presented in Figure 87 and Figure 88 respectively. The voltage estimation has not improved. However, the level of error in estimated voltage angles (Figure 88) and power flows (Figure 89) along the feeder decrease and in most of cases is lower than 8% and 10% respectively. Given that the accuracy of estimation is satisfied no additional meter is needed.
5.2.4 Case study: Measurements available on the first next to the last and the last node

Usually there are no flow measurements available on incoming or outgoing lines in distribution substations; therefore the placement of additional substation sensors is a logical approach.

To identify the next sensor the Step 3 should be done in the case of measurements available on the first and the last node.

The area of the error ellipse at every node is presented in Table 14 and the graphical interpretation of voltage and angle error ellipses is presented in Figure 90. The Blue line presents the case when there are measurements only on the first node and the red when the additional meter is placed. The comparison of $\sqrt{\text{det}P}$, representing the area of the error ellipses in cases of measurement availability on the first and on the first and the last node is presented in Figure 91. If it is to place substation sensor, the recommended place would be the next to the last node, i.e. the node number 11 at the substation 94912, as being the node with the next largest voltage-angle error ellipse.

<table>
<thead>
<tr>
<th>Node</th>
<th>90069</th>
<th>90044</th>
<th>90043</th>
<th>Joint</th>
<th>90625</th>
<th>90862</th>
<th>94356</th>
<th>90638</th>
<th>91143</th>
<th>94192</th>
<th>91045</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{\text{det}P_{x}}$</td>
<td>1.88E-09</td>
<td>6.62E-09</td>
<td>1.1E-08</td>
<td>1.21E-08</td>
<td>1.21E-08</td>
<td>1.62E-08</td>
<td>3.88E-08</td>
<td>5.51E-08</td>
<td>6.79E-08</td>
<td>6.79E-08</td>
<td></td>
</tr>
</tbody>
</table>

Figure 89 Relative errors in flow estimate. Case study: Measurement available at the first node and the last node and with additional power flow measurement at the branch 8-9
Relative errors in voltage and angle estimates are presented in Figure 92 and Figure 93 respectively. The voltage estimation has improved comparing to the situations with measurements on the first and last node (M1 & M12), and to the case with the flow measurement on the branch 8-9. In more than 95% of cases voltage estimation error is less than 0.3%. This is likely to be sufficient considering the statutory voltage requirements.

Comparing to the case M1 and M12, the accuracy of estimated voltage angles has been improved significantly and in more than 95% of cases the errors are less than 10%. However, the improvement is smaller in comparison to the results of the case with the flow measurement.
Figure 94 shows the relative errors of estimated active power flows. In most cases, the errors are relatively small, below 10% while errors at branch 5-6 are much higher due to the use of pseudo measurements.

Figure 92 Relative errors in estimated voltages. Case study: Measurement available on the first, next to the last and the last nodes

Figure 93 Relative errors in angle estimates. Case study: Measurement available on the first, next to the last and the last nodes
Given that accuracy of estimation is satisfied no additional sensor placement is required. Additional power flow measurements will be needed if power flow estimation with higher accuracy is required. It can be concluded that the proposed meter placement approach can reduce significantly the area of the ellipses in the system, and improves the DSSE performance by reducing the error in SE.

5.3 Sensor placement analysis for other EIZ feeders: MERT-E2, BRXB-NE2, AMBL-NW1

We have implemented the same approach for the other EIZ feeders considered in the scope of our study. These include feeder MERT-E2 from Merton EIZ, feeder BRXB-NE2 from Brixton EIZ, and AMBL-NW1. In order to carry out this analysis, we select data from the periods where data are of good quality (no temporary missing data or having high discrepancies). This is summarised as follows:

- BRXB-NE2 study: the 3rd and 4th week of January 2014
- MERT-E2 study: the 3rd and 4th week of February 2014
- AMBL-NW1 study: the 3rd and 4th week of February 2014

The errors of the measurements are taken as defined earlier: 1% for the voltage measurements at primary substations and 2% for the active and reactive power flow measurements, except for AMBL-NW1, where taking into account the specific case of this feeder, the error margin in voltage measurements is assumed to be 6% and 50% for the power measurements.
5.4 Studies on the meter placement along feeder BRXB-NE2

To determine the recommended measurement placement location in feeder BRXB-NE2 (Figure 95), the area of the voltage and angle error ellipse is calculated for every node from the determinant of its error covariance matrix $\sqrt{\text{det} P_x}$, and presented in Table 15. In the base case when there are measurement only on the first node, $\sqrt{\text{det} P_x}$ is highest for the last substation in the feeder, node 8, indicating the new measurement should be placed on its location. Once the measurement is placed on the last substation, according to the $\sqrt{\text{det} P_x}$ form Table 15, column Nodes 1 and 8, the next candidate is identified as node 7. If there are measurements on the Nodes 1, 7 and 8 in Table 15, the next candidate would be node 6.

Table 15 $\sqrt{\text{det}(P_x)}$ for examined case studies with different sensors locations, BRXB-NE2

<table>
<thead>
<tr>
<th>Index</th>
<th>Node Name</th>
<th>$\sqrt{\text{det} P_x}$</th>
<th>Meter location</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>90416</td>
<td>1.14E-08</td>
<td>Node 1</td>
</tr>
<tr>
<td>3</td>
<td>90291</td>
<td>7.43E-08</td>
<td>Node 1 &amp; 8</td>
</tr>
<tr>
<td>4</td>
<td>90415</td>
<td>1.24E-07</td>
<td>Node 1, 7 &amp; 8</td>
</tr>
<tr>
<td>5</td>
<td>90081</td>
<td>2.28E-07</td>
<td>Node 1 &amp; 8, branch flow5-6</td>
</tr>
<tr>
<td>6</td>
<td>90621</td>
<td>2.84E-07</td>
<td>1.50E-09</td>
</tr>
<tr>
<td>7</td>
<td>90153</td>
<td>3.82E-07</td>
<td>2.13E-09</td>
</tr>
<tr>
<td>8</td>
<td>90282</td>
<td>3.91E-07</td>
<td>2.13E-09</td>
</tr>
</tbody>
</table>

Table 16 $\sqrt{\text{det}(P_{ij})}$ for examined case studies with different sensors locations, BRXB-NE2

<table>
<thead>
<tr>
<th>Index</th>
<th>Branch</th>
<th>$\sqrt{\text{det} P_{ij}}$</th>
<th>Meter location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-2</td>
<td>1.27E-09</td>
<td>Node 1</td>
</tr>
<tr>
<td>2</td>
<td>2-3</td>
<td>2.34E-08</td>
<td>Node 1 &amp; 8</td>
</tr>
<tr>
<td>3</td>
<td>3-4</td>
<td>4.06E-08</td>
<td>Node 1 &amp; 8, branch flow5-6</td>
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<td>5.18E-08</td>
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<tr>
<td>5</td>
<td>5-6</td>
<td>5.69E-08</td>
<td>2.13E-09</td>
</tr>
<tr>
<td>6</td>
<td>5-7</td>
<td>5.11E-08</td>
<td>2.13E-09</td>
</tr>
<tr>
<td>7</td>
<td>7-8</td>
<td>1.19E-08</td>
<td>2.13E-09</td>
</tr>
</tbody>
</table>
In order to improve the estimation of power flows, Table 16 shows the optimal location of the power measurements should be put in place. For example, with measurements at node 1 and 8, the optimal additional power flow measurement should be put at branch 5.

It is demonstrated that having sensors on locations 1, 7 and 8 has better effect on voltage quality than having sensors on locations 1, 7 and flow measurement at the branch 5-6. However from the Table 16, it can be seen that flow measurements improve the flow estimates better, in this particular case only upstream of the feeder. It is up to DNO to decide what measurements to choose, regarding their operation practice and needs.

5.5 Studies on the meter placement along feeder AMBL-NW1

![Feeder AMBL NW1 - Queens Park](image)

To determine the recommended measurement placement location in feeder AMBL NW1 (Figure 96), the area of the voltage and angle error ellipse is calculated for every node from the determinant of its error covariance matrix $\sqrt{\text{det} P_x}$, and presented in Table 17. In the base case when there are measurement only at the first node, $\sqrt{\text{det} P_x}$ is highest for the last substation in the feeder, node 6, indicating the new measurement should be placed at this location. Once the measurement is placed at node 6, the next candidate is identified as node number 5. If there are measurements at the Node 1, 5 &6, the next candidate would be node number 4 but as it is a joint then it should be put at node 3.

<table>
<thead>
<tr>
<th>Index</th>
<th>Node Name</th>
<th>$\sqrt{\text{det} P_x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Meter location</td>
<td>Node 1</td>
</tr>
<tr>
<td>1</td>
<td>FDR004</td>
<td>1.34E-07</td>
</tr>
<tr>
<td>2</td>
<td>30166</td>
<td>1.65E-07</td>
</tr>
<tr>
<td>3</td>
<td>30027</td>
<td>1.76E-07</td>
</tr>
<tr>
<td>4</td>
<td>Joint -W0085</td>
<td>1.96E-07</td>
</tr>
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<td>5</td>
<td>30257</td>
<td>2.00E-07</td>
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In order to improve the estimation of power flows, Table 18 shows the optimal location of the power measurements should be put in place. For example, with measurements at node 1 and 6, the optimal additional power flow measurement should be put at branch 2.
Table 18 \(\sqrt{\text{det}(P_{ij})}\) for examined case studies with different sensors locations, AMBL-NW1

<table>
<thead>
<tr>
<th>Index</th>
<th>Branch</th>
<th>Node 1</th>
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<td>4-6</td>
<td>5.70E-09</td>
<td>1.49E-11</td>
<td>1.49E-11</td>
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5.6 Studies on the meter placement along feeder MERT-E2

Figure 97 Feeder MERT-E2

To determine the recommended measurement placement location in feeder MERT-E2 (Figure 97), the area of the voltage and angle error ellipse is calculated for every node from the determinant of its error covariance matrix \(\sqrt{\text{det}P_x}\), and presented in Table 19. In the base case when there are measurement only at the first node, \(\sqrt{\text{det}P_x}\) is highest for the last substation in the feeder, node 19, indicating the new measurement should be placed at this location. Once the measurement is placed at node 19, the next candidate is identified as node number 18. If there are measurements at the Node 1, 18 &19, the next candidate would be node number 16.

Table 19 \(\sqrt{\text{det}(P_x)}\) for examined case studies with different sensors locations, MERT-E2

<table>
<thead>
<tr>
<th>Index</th>
<th>Node Name</th>
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<th>Node 1&amp;18&amp;19</th>
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<td>Joint</td>
</tr>
<tr>
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<td>2.77E-07</td>
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<td>6.77E-08</td>
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</table>
It is important to note that all feeders which have been studied are radial feeder with low branching. Further studies on more complex feeders, with more branching and reconfiguration possibility will be needed in the future. In the case that a reconfiguration of the network is to be studied, the additional load from other feeder can be modelled as the loading of the last node. The power measurement on the last substation would present the flow measurements of the modelled additional load. Therefore, from the above studies, we conclude that in the case of possible reconfiguration of the feeder it would be necessary to have flow measurements at every Normally Open Point (NOP).
6 Conclusion and Recommendations

6.1 Main findings

The key findings from this study can be summarised as follows:

Real world applications of DSSE model have been proven useful and robust

The DSSE approach in combination with pseudo-measurements can be a useful and robust tool for real world applications to improve the monitoring system of distribution networks which is relevant for the present distribution system and critical for smart-grid development. The DSSE model can be applied to:

- improve observability in part of the network where the real measurements are not present; there are conditions where new measurements cannot be installed due to space restriction in the substations, especially underground substations;
- identify faulty measurements, or temporarily bad data due to recording or communication failures for example, so that the bad quality data can be isolated to improve the overall quality of estimation;
- improve the accuracy of the measurements as DSSE takes into account data from all measurements and therefore is able to correct errors from individual measurements;
- carry out general assessment of the meter performances, indicating the ones which should be investigated. For example, in one of our studies the differences between estimated voltage and recorded voltages for substation 94192 are around 0.5% in more than 90% of cases. This clearly indicating that the voltage measurements are underrated. This is in contrast with the factory data specifying the level of error of 0.3% for this type of meter. If the 94192 substation measurements are adjusted and increased by 0.5%, the mismatch between the estimates and corrected measured is less than 0.1% for more than 90% of cases;
- minimise the cost of monitoring system by reducing the number of real measurements. This is particularly important for distribution networks where the number of nodes is significantly greater than in transmission.

Further work will be required to investigate and quantify the economic benefits of rolling out the application of DSSE in the UK power distribution networks, but as can be preliminary concluded from our studies that the implementation of DSSE will have positive impacts on most of DNOs’ business activities, e.g. daily operation, planning, asset management, safety, and network pricing activities. The benefits of DSSE in reducing the number of measurements required also provide strong business cases for its implementation.

With regards to the accuracy of the DSSE model, our studies suggest that the uncertainty (error margin) of the estimated voltages is relatively small and in most of the cases, the error margin is less than 0.22% (in comparison to the error margin of individual meters: 0.3%-0.6%). We also observe that by having the right measurements at substations, the error margin of the estimated power flow can be reduced to typically below 10%. The use of state estimation can also enhance network visibility for the lateral sections where the substations have no measurements installed. The results of the studies are summarised in Table C. 1 and Table C. 2 in Appendix C: Uncertainty in Estimated Voltages and Power Flows.
Identified practical issues and challenges in the implementation of DSSE on the EIZ feeders

We have observed that insufficient network coverage, measurements data and network parameters quality and availability, can cause that state estimation algorithm may fail to provide useful results. For example:

- In terms of recorded measurements there is insufficient quantity and quality of measurements. The V, P and Q measurements are unavailable at the supply points. There are only records of current measurements at the supply points of 3 feeders. Moreover, Queen’s Park EIZ does not have any primary substation recordings.
- The measurements on secondary substations are available on 24/34 EIZs locations. However, only 21/34 can be trusted.

We also observe that the way in which the available network data are recorded and stored in DNO database can affect the effectiveness and accuracy of SE. For example:

- Some network databases only provide information associated with the first section of the HV feeders, although the rest of the sections could involve cables of different types, having different electrical characteristics. The impedances of the line connecting two busses are calculated according to the first segment of the feeder which may introduce error in the network parameters and eventually in the DSSE calculations;
- Time synchronisation in taking the measurement samples at different measurement locations is very important for the implementation and application of SE, and this is an area that will require further development.
- Bad data and missing data during some time periods can additionally contribute to the inaccuracy of SE. This is mainly the case with the measurements form the new RTUs, which is probably just installation and data collection issues;
- Synergies between substation sensors and smart meters could not be used at present, since only a small number of customers at the considered feeders are instrumented.

We have also identified the challenges in implementing DSSE in an unbalanced three phase distribution system. Unlike in transmission system where the loading across different phases is practically balanced, the level of imbalance in distribution network can be large, which would reduce the accuracy of the DSSE which assumes the system is balanced. In the developed DSSE model, the single phase positive sequence equivalent circuit is used to model the networks under study assuming steady state balanced conditions. While this is a limitation of the current model, applying DSSE to an unbalanced system will also not be possible in this project taking into account that only phase values of currents and voltages are recorded on the old RTU substations, but not power factor or active and reactive power per phase, and there are no such records on the primary substations.

Meter placement methodology has been implemented successfully on the selected EIZ system

The proposed and developed meter placement methodology in EIZs is robust and its applications have been demonstrated. We have also demonstrated that with a relatively small number of real measurements the level of network visibility can be improved significantly. For example:

- The high quality of voltage estimates of Feeder BRXB-SE3 can be achieved only with 2 sensors – on the beginning and on the end of the feeder. However, it should be taken in account that this particular feeder is relatively short and lightly loaded, and even if the loading is 50% higher than
the maximum recorded peak within the considered year, the voltage drop from the beginning to the end of the feeder is still very small and equal to 0.8%. On the other hand, the accurate voltage magnitude estimation does not guarantee good active line flow estimation. The farther from the measured nodes, the level of estimation errors typically increases.

- Placing meters in the end of the feeder is of strategic significance not only for estimation in business as usual situations, but specially in the cases of network reconfiguration when there is a need of the parts of other feeders to be supplied as well. This is especially important if the feeders are tapered and there is need to avoid overloading the particular sections.
- It is important to understand the DNO’s objectives in sensors placement, if it is improvement of voltages and/or flow estimation accuracy or overall state of the network. If there are some parts of the network more important than others, their observability is of higher importance, and instrumentation of that part could be more preferred.
- If the flow in the branch towards substation 90625 in Brixton zone, which is off main line, was important for the operational purposes of the distributions system, the sensors would be placed there rather than on the other location. In the analysed case it stays unmonitored since having meters there doesn’t contribute a lot to overall network observability. However the accuracy in estimates of voltage at that node is not compromised by lack of the sensor.

6.2 Recommendations

Based on the experience gained in this project, we list a set of recommendations that will assist implementation and application of DSSE in distribution networks. The implementation of DSSE as an integral part of the DMS monitoring system will enhance the capability of the distribution network operator to make informed operation decisions so that the network can be operated securely while making full use available network capacity. To start with, it is recommended that DSSE is employed in stressed parts of distribution networks that are operated close to the operating voltage and thermal limits, and then be subject to close monitoring. In order to facilitate the application of DSSE, additional measurements may need to be installed. The approach described in this report can be used to determine the locations and types of measurements needed. Enhancing system state visibility will also benefit network planners as they can identify the changes in the utilisation patterns and devise optimal strategy for network reinforcement based on the information gathered from the monitoring system.

It is important to highlight that the main barriers for effective implementation of state estimation lie in the availability and quality of network data. Improvement and standardisation of measurement and recording practice as well as further enhancement of the DSSE algorithm will contribute to more effective and efficient distribution network system monitoring and control. In this context, our recommendations can be summarised as follows:

- Synchronising readings of all measurement points - the measurements at secondary substations are recorded half-hourly while primary measurements are recorded as snapshots at irregular intervals. Therefore it would be good to have primary measurements recorded as rms average values with the same sampling rate as the rest of the feeder or often. Sampling rate of new type RTUs is 10 minutes which would improve network state estimation.
• Checking the accuracy of the recorded network parameters especially the ones that consist of multiple cables and the accuracy of recorded transformer winding ratios and the tap-changing positions;

• Validating the measurements accuracy of the sensors in the key feeders where the DSSE is to be applied for operational purposes. It is important to highlight that the sensor accuracy is a key factor affecting the outcome of the DSSE. A robust procedure of RTUs installation and commissioning needs to be implemented;

• Improving the availability of key measurements. For example, ones of important measurements are those at the beginning of each feeder; making them available and avoiding the need for pseudo measurements, could crucially improve the estimation. Queen’s Park EIZ does not have any primary substation recordings and their readings are necessary for adequate DSSE application;

• The use of typical load profiles of connected customers in the applications of pseudo-measurements for unmonitored substations is recommended since it can significantly improve the accuracy of the DSSE model;

• As a rule of thumb, placing measurements at feeder supply point at primary substation and towards the end of important feeder branches would enhance the accuracy of voltage estimation and enhance bad data detection. It is also recommended to have sensors at every NOP and the power flow measurements at the sections of the network which are heavily loaded;

• As the network imbalance impacts the accuracy of DSSE, it may be appropriate to consider developing DSSE specifically for unbalanced 3 phase networks. Application of techniques for improving phase balance that would reduce network losses and enhance the utilisation of LV and HV networks would also enhance the accuracy of state estimation; and

• A further theoretical development of DSSE and meter placement techniques and algorithms is also needed to address the limitation of the current approaches in dealing with actively controlled network re-configuration; and

• Carrying out comprehensive studies analysing and quantifying in more detail the cost and benefits for rolling out the applications of DSSE and to establish standards for its implementation.
7 References

## Appendix A

### Table A. 1 Customer profile class table

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</tr>
<tr>
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<td>Non-Domestic Unrestricted</td>
</tr>
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<td>Non-Domestic Economy 7</td>
</tr>
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<td>Non-Domestic Maximum Demand 0-20% Load Factor</td>
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<tr>
<td>6</td>
<td>Non-Domestic Maximum Demand 20-30% Load Factor</td>
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<td>Non-Domestic Maximum Demand 30-40% Load Factor</td>
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<td>8</td>
<td>Non-Domestic Maximum Demand &gt;40% Load Factor</td>
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</table>
Appendix B: Estimated Power Flows

In this section, we present and discuss the results of our study estimating the power flows along the EIZ feeders based on the one year measurement data obtained from the LCL project. The uncertainty of the estimated active power flows has also been analysed and discussed.

B.1 Uncertainty of the estimated power flows along Feeder BRXB-SE3 from Brixton EIZ

The following figures describe the uncertainty in the estimated active power flows for all branches of the feeder BRXB-SE3 based on the results from the DSSE model run and the measurement data across one year period. The period of observation is from 01 March 2013 to 01 March 2014.

Uncertainty of half-hourly estimated active power flows for the first four branches before the joint point is modest, i.e. less than 2% for almost all year as presented in Figure B. 1. This is driven by the availability and the accuracy of the real measurements at the first 4 nodes of which these branches connect.

Figure B. 1 Annual active flow uncertainty for the first four branches of the feeder BRXB-SE3
Figure B. 2 demonstrates the uncertainty of estimated active power flow for branch 5-6, from the joint to the unmonitored substation 90625. For substations with pseudo measurements (50% error margin), the average uncertainty is similar, i.e., 46%.

Figure B. 2 Year-round analysis on the uncertainty of the estimated active power flows for branch 5-6, from the joint to the unmonitored substation 90625.

The uncertainty of the estimated active power flow of the branches connected to the unmonitored nodes is mainly up to 10 %, Figure B. 3, while the last two branches which connect monitored substations have mainly low (less than 4%) uncertainty as presented in Figure B. 4.

Figure B. 3 Annual active flow uncertainty for the branches connected to the unmonitored nodes of the feeder BRXB-SE3
Figure B. 4 Year-round analysis on the uncertainty of the estimated power flow for branch 10-11 and branch 11-12

It is important to note that even the uncertainty of estimated power flows may be large at certain branches, the impact may be very limited. Bearing in mind that the HV network is designed to meet the security standard N-1, the capacity of each feeder is likely to be double than its maximum load under normal operating condition. For example, the cables ratings under normal conditions are at least 280 A and the maximum recorded current at the supply point of the feeder is only 131 A, there is no possibility of reaching thermal limits of this feeder.

B.2 Uncertainty of the estimated power flows along Feeder BRXB-NE2 from Brixton EIZ

Figure B. 5 and Figure B. 6 show the uncertainty of the estimated active power flows for all branches of the feeder BRXB-NE2, as results from the DSSE model run across one year period, from 01 March 2013 to 01 March 2014. The uncertainty of half-hourly estimated active power flows for the first branch is almost always less than 1.41%. Going further down the feeder, the level of uncertainty is increasing. In most cases, the uncertainty of the estimated flows for the last branch is less than 4% since the last substation has measurements. This is demonstrated in Figure B. 6.
Figure B. 5 Year round analysis of the uncertainty of the estimated power flows for the branches of the feeder BRXB-NE2
Figure B.6 Year round analysis of the uncertainty of the estimated power flows for the last branch of the feeder BRXB-NE2

The maximum recorded current for the examined year period on the supply point of the feeder was 170 A. The cable rating of the first two branches is 325 A and for the last two 240 A. Taking in account uncertainty of the flow estimates, that this is radial feeder and that flow decreases down the feeder, reaching thermal limits of this feeder is not likely.

B.3 Uncertainty of the estimated power flows along Feeder AMBL-NW1 from Queen’s Park EIZ

The uncertainty of the estimated active power flows for all branches of the feeder AMBL-NW1, as results from the DSSE model run across one year period, from 01 March 2013 to 01 March 2014, is presented in Figure B. 7. The uncertainty of half-hourly active flow estimate for the first branches is almost always less than 2%, while further down the feeder, the uncertainty of estimated power flows increases to 7%. 
Figure B. 7 Year round analysis of the uncertainty of the estimated power flows for the feeder AMBL-NW1

B.4 Uncertainty of the estimated power flows along Feeder MERT-E2 from Merton EIZ

The following figures describe the uncertainty in the estimated active power flows for all branches along the feeder MERT-E2, as results from the DSSE model run across one year period. However, for few operating snapshots, data are incomplete and therefore excluded from the analysis. The period of
The observation is from 01 March 2013 to 01 March 2014. The results of our studies are shown in Figure B. 8.

The uncertainty of the half-hourly active power flows for the first three branches before the first unmonitored substation (8560), is less than 2% in all cases. Going down the feeder, the uncertainty increases. In most cases, the error margin is below 10% for the main branches. However, the uncertainty of the lateral sections, which connect to unmonitored substations, is considerably high, i.e. in average around 55%.
It is important to note, if the cable rating is large enough to deal with the uncertainty of the estimated power flows, it may not necessarily to install power flow measurements. However, for the heavily loaded cable and if there is a concern that thermal limit may be reached, the power flow measurement becomes critical.
Appendix C: Uncertainty in Estimated Voltages and Power Flows

Table C.1 Uncertainty in estimated voltages

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<th>Measurement Uncertainty [%]</th>
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<th>Node Name</th>
<th>AMBL-NW1 Uncertainty in estimation [%]</th>
<th>Measurement Uncertainty [%]</th>
<th>MERT-E2 Node Index</th>
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Project Overview

Low Carbon London, UK Power Networks’ pioneering learning programme funded by Ofgem’s Low Carbon Networks Fund, has used London as a test bed to develop a smarter electricity network that can manage the demands of a low carbon economy and deliver reliable, sustainable electricity to businesses, residents and communities.

The trials undertaken as part of LCL comprise a set of separate but inter-related activities, approaches and experiments. They have explored how best to deliver and manage a sustainable, cost-effective electricity network as we move towards a low carbon future. The project established a learning laboratory, based at Imperial College London, to analyse the data from the trials which has informed a comprehensive portfolio of learning reports that integrate LCL’s findings.

The structure of these learning reports is shown below:

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